REFOR algorithm of the dynamic parametrical modeling on Multi-Degree of Freedom systems

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Abstract: In allusion to the identification problem of Multi-Degree of Freedom (MDOF) nonlinear systems, a new algorithm of systems' dynamic parametrical modeling, called the Redundant Extended Forward Orthogonal Regression (REFOR) is developed in this study, to construct a response position-dependent nonlinear dynamic parametrical model. REFOR is a new algorithm for dynamic parametrical model, which aims to avoid missing some significant model terms when using Extended Forward Orthogonal Regression (EFOR) algorithm. Based on Non-linear Autoregressive with Exogenous inputs (NARX) model of Single Input Single Output (SISO), corresponding to different response position, a common model structure is built via REFOR and a function relationship between unified model term's coefficients and position parameters is established. A dynamic parametrical model of MDOF nonlinear systems is constructed as a consequence. Further, a SIMO 5DOF nonlinear system is taken as a case study to clarify the advantage of REFOR and its application in modeling. Finally, a dynamic parametrical model of cantilever beam is established from experimental data. The results indicate that the dynamic parametrical model of nonlinear systems, which based on REFOR, can accurately predict output response, which provide a theoretical basis for the optimal design of SIMO nonlinear systems' modeling methods.

Key-Words: MDOF, dynamic parametrical model, NARX model, REFOR algorithm, cantilever beam.

1 Introduction

In practice, systems have to be described by using a mathematical model for the purpose of analysis and design [1-2]. As a class of numerical model, Non-linear Autoregressive with Exogenous inputs (NARX) model is mainly used to describe the nonlinear discrete system [3]. The structure of NARX model is simple and the model has been widely used in many applications, such as industrial control, target tracking, etc. However, the coefficients of NARX model terms don't have any physical meaning, which means it's difficult to make further analysis on the underlying system.

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Because of the problem mentioned above, the dynamic parametrical model based on NARX was introduced, in which the physical parameters are explicitly expressed. Tong [4] introduced a class of models called threshold models. The models consist of linear models where the coefficients of model terms have functional relation with the amplitude of past values of some variable (observation position), the associated estimation algorithms have also been used in modeling process and the method has been validated.

Relevant algorithms have been referred as the dynamic parametrical model been introduced (VOON [5]; Cyrot-Normand, [6]). Haber [7] derived algorithms for the identification of linear models with signal-dependent parameters to represent non-linear systems. Diekmann [8] introduced an on-line algorithm and showed that the different sets of parameters in operating-point-dependent linear models can be estimated in parallel from one period of measurement. Based on the NARX model, Wei [9] built a dynamic parametrical model for Single Input Single Output (SISO) systems, where the particle damper system was addressed. In Wei's study, the dynamic parametrical model was identified by using the Extended Forward Orthogonal Regression (EFOR) algorithm, which is efficient but in order to obtain the common structure of the final model, some terms that may contain important information have to be ignored. This may induce errors in the analysis and design of the systems.

To address this issue, in the present study, a new algorithm known as the Redundant Extended Forward Orthogonal Regression (REFOR) is proposed to identify the dynamic parametrical model of nonlinear systems, where the multiple output system subject to a single input is discussed. A 5DOF nonlinear system is studied as an example to illustrate the advantage of the newly proposed algorithm compared with the traditional EFOR approach. Moreover, considering the application of engineering practice, an experiment is conducted on a cantilever beam and the algorithm is applied to build the dynamic parametrical model of the beam system.

The paper is organized as follows. Section 1 briefly reviews the dynamic parametrical model of nonlinear systems. Section 2 introduces the new modeling method of dynamic parametrical model with REFOR algorithm, an illustrative simulation example and experimental validation are considered to test the new method then. Conclusions are finally drawn in Section 3.

2 Modeling method based REFOR

2.1 Dynamic parametrical model

NARX model of nonlinear systems

As a new representation for a wide class of nonlinear systems, the NARX model was firstly introduced in 1981[3]. The most commonly used NARX model is the power-form polynomial representation

$$y(t)=\theta_0+\sum_{i_1=1}^{n}\theta_{i_1}x_{i_1}(t)+\sum_{i_1=1}^{n}\sum_{i_2=i_1}^{n}\theta_{i_1i_2}x_{i_1}(t)x_{i_2}(t)+\cdots+\sum_{i_1=1}^{n}\cdots\sum_{i_l=i_{l-1}}^{n}\theta_{i_1i_2\cdots i_l}\prod_{k=1}^{l}x_{i_k}(t) \quad (1)$$

$$x_m(t)=\begin{cases}u(t-m) & 1\le m\le n_u\\ y(t-(m-n_u)) & n_u+1\le m\le n_u+n_y\end{cases} \quad (2)$$

where $y(t)$ and $u(t)$ are the system output, input sequences; n_y and n_u are the maximum lags for the system output, input ; $n=n_u+n_y$; l is the degree of polynomial nonlinearity, which means the highest order among the terms;

Note that (1) can be expressed using a generic linear-in-the-parameters representation

$$y(t)=\sum_{m=1}^{M}\theta_m p_m(t) \quad (3)$$

where $p_m(t)$ with $m=1,2,\cdots,M$ are the regressors formed by some combinations of predetermined model variables chosen from the vector $\boldsymbol{x}(t)=[1,u(t-1),\cdots,u(t-n_u),y(t-1),\cdots,y(t-n_y)]$; θ_m are the model coefficients; M is the total number of potential model terms in the NARX model and $M=\frac{(n+l)!}{n!l!}$.

Structure of dynamic parametrical model

The structure of NARX model may not identical, corresponding to the different cases of parameter properties [10]. In order to construct an overall dynamic parametrical model of nonlinear systems, the objective is to unify the models structure through REFOR algorithm, corresponding to K different cases of parameter properties. It is assumed that all the K data sets can be represented using a common model structure, denote the kth NARX model can be represented by the regression model below

$$y_k(t)=\sum_{m=1}^{M_0}\theta_{k,m}p_m(t) \quad (4)$$

where $\theta_{k,m}$ is the mth coefficients of the kth NARX model, and it has some function relation with physical parameter R; M_0 is the total number of model terms in the common model and $M_0\le M$.

Model (4) shows that the structure of dynamic parametrical model is based on NARX model, while the coefficients get parameterized that have function relation with physical parameter.

2.2 Evaluation of dynamic parametrical model

REFOR

The associated algorithm with dynamic parametrical model, REFOR, is an optimal of traditional EFOR. Based on the core idea of Forward Regression Orthogonal Least Squares (FROLS) algorithm, REFOR firstly builds a series of NARX models, corresponding to different cases of parameter properties, under the premise of a common model structure, including terms of each model as comprehensive as possible.

The basic idea of the REFOR algorithm is to find a common structure for all K NARX models such that the coefficient of model terms can be formulated as a function of physical parameters. The above REFOR algorithm can be summarized as follows.

Procedure 1 Orthogonalization of models

A compact matrix form of kth ($k = 1, \cdots, K$) NARX model is.

$$\boldsymbol{y}_k = \boldsymbol{P}_k \boldsymbol{\theta}_k \tag{5}$$

where $\boldsymbol{y}_k$ is the output vector of kth model; $\boldsymbol{\theta}_k$ is the parameter vector of kth model; $\boldsymbol{P}_k$ is a matrix consists of candidate model terms of kth model.

$$\boldsymbol{P}_k = [\boldsymbol{p}_{k,1}, \boldsymbol{p}_{k,2}, \cdots, \boldsymbol{p}_{k,M}] = \begin{bmatrix} p_{k,1}(p) & p_{k,2}(p) & \cdots & p_{k,M}(p) \\ p_{k,1}(p+1) & p_{k,2}(p+1) & \cdots & p_{k,M}(p+1) \\ \vdots & \vdots & \ddots & \vdots \\ p_{k,1}(N) & p_{k,2}(N) & \cdots & p_{k,M}(N) \end{bmatrix} \tag{6}$$

Assume that the regression matrix $\boldsymbol{P}_k$ is full rank in columns, and there exists matrix $\boldsymbol{W}_k$ and $\boldsymbol{A}_k$ such that can be orthogonally decomposed as [3].

$$\boldsymbol{P}_k = \boldsymbol{W}_k \boldsymbol{A}_k \tag{7}$$

where $\boldsymbol{A}_k$ is an $M \times M$ unit upper triangular matrix and $\boldsymbol{W}_k$ is an $(N - p + 1) \times M$ matrix with orthogonal columns.

The space spanned by the orthogonal vector $[\boldsymbol{\omega}_{k,1}, \boldsymbol{\omega}_{k,2}, \cdots, \boldsymbol{\omega}_{k,M}]$ is the same as that spanned by the basis vector $[\boldsymbol{p}_{k,1}, \boldsymbol{p}_{k,2}, \cdots, \boldsymbol{p}_{k,M}]$, and (1) can be expressed as.

$$\boldsymbol{y}_k = (\boldsymbol{P}_k \boldsymbol{A}_k^{-1})(\boldsymbol{A}_k \boldsymbol{\theta}_k) = \boldsymbol{W}_k \boldsymbol{G}_k \tag{8}$$

where $\boldsymbol{G}_k = \boldsymbol{A}_k \boldsymbol{\theta}_k = [g_{k,1}, g_{k,2}, \cdots, g_{k,M}]^{\mathrm{T}}$; $g_{k,m} (m = 1, 2, \cdots M)$ is the orthogonal coefficients.

Orthogonalize the model term vector of matrix via the basis of Gram-Schmidt algorithm [3].

$$\boldsymbol{\omega}_{k,m} = \boldsymbol{p}_{k,m} - \sum_{i=1}^{m-1} \frac{< \boldsymbol{p}_{k,m}, \boldsymbol{\omega}_{k,i} >}{< \boldsymbol{\omega}_{k,i}, \boldsymbol{\omega}_{k,i} >} \boldsymbol{\omega}_{k,i} \tag{9}$$

$$g_{k,m} = \frac{< \boldsymbol{y}_k, \boldsymbol{\omega}_{k,m} >}{< \boldsymbol{\omega}_{k,m}, \boldsymbol{\omega}_{k,m} >} \tag{10}$$

Procedure 2 Modeling method of REFOR

Step1: Let $\boldsymbol{\omega}_{k,m}^{(1)} = \boldsymbol{p}_{k,m}$, according to (10), calculate

$$g_{k,m}^{(1)} = \frac{< \boldsymbol{y}_k, \boldsymbol{\omega}_{k,m}^{(1)} >}{< \boldsymbol{\omega}_{k,m}^{(1)}, \boldsymbol{\omega}_{k,m}^{(1)} >} \tag{11}$$

where superscript (1) denote the first step of the algorithm.

For $m = 1, 2, \cdots M$, calculate the Error Reduction Ratio (ERR) according to [11] as

$$ERR_{k,m}^{(1)} = \frac{(g_{k,m}^{(1)})^2 < \boldsymbol{\omega}_{k,m}^{(1)}, \boldsymbol{\omega}_{k,m}^{(1)} >}{< \boldsymbol{y}_k, \boldsymbol{y}_k >} \times 100\% \tag{12}$$

where ERR was introduced to measure the significance of the model terms in the description of system. Denote

$$l_{k,1} = \arg\max \left\{ \mathrm{ERR}_{k,m}^{(1)} \right\}, 1 \le m \le M \tag{13}$$

such that $\mathrm{ERR}[l_{k,1}] = \max \left\{ \mathrm{ERR}_{k,m}^{(1)} \right\}, 1 \le m \le M$, the first significant basis can then be selected and the first associated orthogonal vector is chosen as $\boldsymbol{\omega}_{k,1} = \boldsymbol{\omega}_{k,l_{k,1}}^{(1)}$ $(k = 1, \cdots, K)$. Specify a new vector $\boldsymbol{\alpha}_{k,m} (1 \le m \le M)$, which representing the selected term and let $\boldsymbol{\alpha}_{k,1} = \boldsymbol{p}_{k,l_{k,1}}$.

Step *s* (*s*>=2): With regard to the selected orthogonal vectors $\boldsymbol{\omega}_{k,1}, \boldsymbol{\omega}_{k,2}, \cdots \boldsymbol{\omega}_{k,S-1}$, let $m \ne l_{k,1}, l_{k,2}, \cdots, l_{k,S-1}$, calculate individually.

$$\boldsymbol{\omega}_{k,m}^{(s)} = \boldsymbol{p}_{k,m} - \sum_{l=1}^{s-1} \frac{< \boldsymbol{p}_{k,m}, \boldsymbol{\omega}_{k,l} >}{< \boldsymbol{\omega}_{k,l}, \boldsymbol{\omega}_{k,l} >} \boldsymbol{\omega}_{k,l} \tag{14}$$

$$g_{k,m}^{(s)} = \frac{< \boldsymbol{y}_k, \boldsymbol{\omega}_{k,m}^{(s)} >}{< \boldsymbol{\omega}_{k,m}^{(s)}, \boldsymbol{\omega}_{k,m}^{(s)} >} \tag{15}$$

$$\mathrm{ERR}_{k,m}^{(s)} = \frac{(g_{k,m}^{(s)})^2 < \boldsymbol{\omega}_{k,m}^{(s)}, \boldsymbol{\omega}_{k,m}^{(s)} >}{< \boldsymbol{y}_k, \boldsymbol{y}_k >} \times 100\% \tag{16}$$

Let

$$l_{k,s} = \arg\max \left\{ \mathrm{ERR}_{k,m}^{(s)} \right\} \tag{17}$$

Let $\boldsymbol{\omega}_{k,s} = \boldsymbol{\omega}_{k,l_{k,s}}^{(s)}$ and $\boldsymbol{\alpha}_{k,s} = \boldsymbol{p}_{k,l_{k,s}}$.

Step *s*+1: The search is terminated at the M_k step when the Error Reduction Ratio (ESR) is less than a pre-specified threshold

$$\mathrm{ESR} = 1 - \sum_{i=1}^{M_k} \max \left\{ \mathrm{ERR}_k^{(i)} \right\} \le \rho \tag{18}$$

The final *k*th NARX model is the linear combination of the M_k significant terms selected from the *M* candidate terms

$$\boldsymbol{y}_k = \sum_{m=1}^{M_k} g_{k,m} \boldsymbol{\omega}_{k,m} \tag{19}$$

Step *s*+2: There will be a total of *K* orthogonal type of NARX models like (19) after the selection process above, and all of the terms that each model identifies should be brought into the final common-structured model via REFOR.

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$$\boldsymbol{y}_k = \sum_{m=1}^{M_0} g_{k,m}\boldsymbol{\omega}_m \\ \max\{M_1, M_2, \cdots, M_K\} \le M_0 \le M_1 + M_2 + \cdots + M_K \tag{20}$$

where M_0 is the total number of terms in the final model, $M_1, \cdots, M_K$ is the total number of terms that each model selects.

Step s+3: Because of the new added terms, the coefficients of them need to be re-estimated. Assume a subset Φ'_k, which consists of orthogonal vectors

$$\Phi'_k = \left\{\boldsymbol{\omega}_{k,1}, \boldsymbol{\omega}_{k,2}, \cdots, \boldsymbol{\omega}_{k,M_k}\right\} \tag{21}$$

A subset which consists of new added vectors is defined as

$$\hat{\Phi}_k = \left\{\boldsymbol{p}_{k,t_1}, \boldsymbol{p}_{k,t_2}, \cdots, \boldsymbol{p}_{k,t_{M_0-M_k}}\right\} \tag{22}$$

where $\Phi'_k \subseteq \boldsymbol{W}_k$, $\hat{\Phi}_k \subseteq \boldsymbol{P}_k$, $t_1, t_2, \cdots t_{M_0-M_k}$ denotes the position of new added terms.

Let $\boldsymbol{\omega}_{k,t_1} = \boldsymbol{p}_{k,t_1}, \cdots, \boldsymbol{\omega}_{k,t_{M_0-M_k}} = \boldsymbol{p}_{k,t_{M_0-M_k}}$, calculate.

$$g_{k,t_m} = \frac{<\hat{\boldsymbol{y}}_k, \boldsymbol{\omega}_{k,t_m}>}{<\boldsymbol{\omega}_{k,t_m}, \boldsymbol{\omega}_{k,t_m}>} \\ m = 1, \cdots, M_0 - M_k \tag{23}$$

Define $\hat{\boldsymbol{y}}_k = \hat{\Phi}_k\hat{\boldsymbol{G}}_k = \boldsymbol{y}_k - \Phi'_k\boldsymbol{G}'_k = \boldsymbol{y}_k - \sum_{m=1}^{M_k}\boldsymbol{\omega}_{k,m}g_{k,m}$ as error sequence of kth NARX model;

Note that from (23) that all of coefficients of new added terms can be re-estimated, the remaining $K-1$ NARX models will be revised when repeat the modeling process above.

Procedure 3 Unify the structure of dynamic parametrical model

Note that the model (20) is not the final dynamic parametrical model. As for original model terms, the coefficients need to be inverse orthogonalized based on Inverse-Gram-Schmidt algorithm [3].

$$\begin{cases} \theta_{k,M_k} = g_{k,M_k} \\ \theta_{k,M_k-1} = g_{k,M_k-1} - a_{M_k-1,M_k}\theta_{k,M_k} \\ \theta_{k,M_k-2} = g_{k,M_k-2} - a_{M_k-2,M_k-1}\theta_{k,M_k-1} - a_{M_k-2,M_k}\theta_{k,M_k} \\ \vdots \\ \theta_{k,m} = g_{k,m} - \sum_{i=m+1}^{M_k} a_{m,i}\theta_{k,i} \end{cases} \tag{24}$$

where

$$a_{m,i} = \frac{<\boldsymbol{\alpha}_{k,i}, \boldsymbol{\omega}_m>}{<\boldsymbol{\omega}_m, \boldsymbol{\omega}_m>}, \quad 1 \le m \le i-1 \\ m = M_k - 1, M_k - 2, \cdots, 1 \tag{25}$$

As for new added terms, the coefficients will be.

$$\begin{gathered}\theta_{k,m} = g_{k,m} \\ m = M_k + 1, \cdots, M_0\end{gathered} \tag{26}$$

After the transformation of $g_{k,m}$, the sum of K NARX models will be established, which have unified-structure. Then the relationship between $\theta_{k,m}$ and parameter property need to be built through mathematical expression [12].

This paper adopts the polynomial expression to describe the relationship between $\theta_{k,m}$ and R.

$$\theta_{k,m}(\boldsymbol{R}) = \sum_{j_1=0}^{J} \cdots \sum_{j_q=0}^{J} \beta_{j_1,\ldots,j_q} R_1^{j_1} R_2^{j_2} \cdots R_q^{j_q} \tag{27}$$

where J is the degree of polynomial nonlinearity; $R_m (m = 1, \cdots, q)$ denotes the position parameter; $\beta_{j_1,\ldots,j_S}$ is the coefficient of polynomial term, which can be estimated by Least Squares (LS) algorithm [13].

The final dynamic parametrical model concerned with position will be established, which is consists of (4) and (27).

2.3 A case study and discussion

Consider an SIMO 5DOF nonlinear system in Fig. 1. The five lumped masses are $m_1 = m_2 = m_3 = m_4 = m_5 = 1\text{kg}$. The five masses are connected by series of springs and dampers, where linear springs with stiffness $k_1 = k_2 = k_3 = k_4 = k_5 = 10^7\,\text{N/m}$, nonlinear springs with stiffness $k_{non} = 10^{11}\,\text{N/m}^2$, linear dampers with damping coefficients $c_1 = c_2 = c_3 = c_4 = c_5 = 10\,\text{Ns/m}$ and nonlinear dampers with damping coefficients $c_{non} = 5000\,\text{Ns}^2/\text{m}^2$.

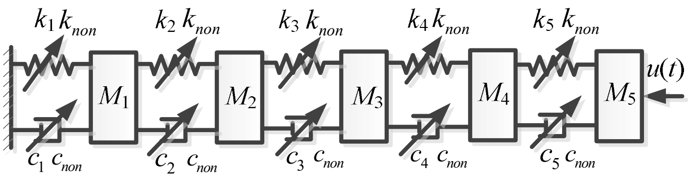

Fig.1. An SIMO 5DOF nonlinear system

Construct the NARX models correspond to mass 1,3,4,5. In order to stimulate the system, Gaussian processes with zero mean and a variance $\sigma^2 = 4^2$ were used as the external input. The system was simulated using a fourth-order Runge-Kutta method, each data set consists of 3000 data pairs of the input (applied force) and the output (acceleration) observations, sampled with a frequency $f_s = 1000\text{Hz}$. Four data sets, corresponding to mass 1,3,4,5, with the same input, as shown in Fig.2, but with different output, as shown in Fig.3, were used for model identification, and one data set, corresponding to mass 2, was used to test the performance of the identified dynamic

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parametrical model.

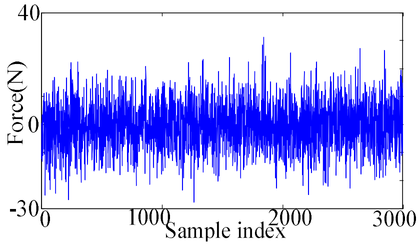

Fig.2. The input signal used for nonlinear system

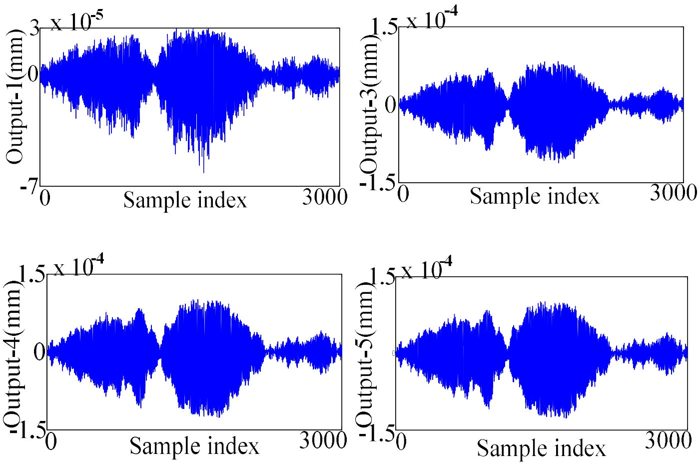

Fig.3. The output signals corresponding to mass 1,3,4,5

Denote the system input and output sequence using $\{u(t)\}_{t=1}^{N}$ and $\{y(t)\}_{t=1}^{N}$, respectively, with $N = 3000$. The predictor vector for common-structured models was chosen to be $\boldsymbol{x}(t) = [x_1(t), \cdots, x_{10}(t)]^{\mathrm{T}}$, where $x_k(t) = u(t-k)$ for $k = 1, \cdots, 5$ and $x_k(t) = y(t-k+5)$ for $k = 6, \cdots, 10$, degree of model was chosen to be $l = 2$. The candidate model involves a total of $M = \dfrac{(n_u + n_y + l)!}{(n_u + n_y)! l!} = 66$ candidate model terms. Initial candidate common-structured NARX model for all the four data sets was defined below

$$y(t) = \theta_0 + \sum_{i_1=1}^{10} \theta_{i_1} x_{i_1}(t) + \sum_{i_1=1}^{10} \sum_{i_2=i_1}^{10} \theta_{i_1 i_2} x_{i_1}(t) x_{i_2}(t) \tag{28}$$

Firstly, there are four NARX models corresponding to mass 1,3,4,5, which were identified by REFOR algorithm. Denote the pre-specified threshold $\rho = 0.05$ and the results were given in Table 1.

Table 1. Identification result for each mass

Search step	Mass 1			Mass 3		
	Terms	ERRs	Coefficients	Terms	ERRs	Coefficients
1	$y(t-4)$	63.7831	-0.935	$y(t-4)$	72.9887	-0.9352
2	$y(t-5)^2$	14.9875	-4.6026×10^3	$y(t-5)^2$	14.5148	-1.911×10^3
3	$y(t-3)$	5.7733	-0.3756	$y(t-5)$	6.4915	0.7857
4	$y(t-3)^2$	6.2793	-1.0056×10^4	$y(t-1)$	3.5658	0.6097
5	$y(t-4)^2$	1.3041	-1.352×10^4			
6	$y(t-3)y(t-4)$	1.8292	6.3501×10^3			
7	$u(t-2)$	0.5415	1.3663×10^{-7}			
8	$y(t-3)y(t-5)$	0.4921	8.2201×10^3			
9	$y(t-1)$	0.5165	-0.1399			
Total		95.5066			97.5608	
Search step	**Mass 4**			**Mass 5**		
	Terms	**ERRs**	**Coefficients**	**Terms**	**ERRs**	**Coefficients**
1	$y(t-4)$	74.2974	-0.9384	$y(t-4)$	74.4698	-0.9548
2	$y(t-5)^2$	12.9988	-1.5788×10^3	$y(t-5)^2$	11.8168	-1.7486×10^3
3	$y(t-5)$	7.3656	0.802	$y(t-5)$	7.3618	0.7337
4	$y(t-1)$	3.6649	0.6201	$y(t-1)$	2.9779	0.5417
Total		98.3267			96.6263	

Under the premise of common-structured model, REFOR then includes terms of each NARX model as comprehensive as possible, and the final 10 selected common model terms were shown in Table 2.

Table 2. Identification result by using REFOR

No.	Terms	Coefficient for different data sets			
		Mass 1	Mass 3	Mass 4	Mass 5
1	$u(t-2)$	1.3663×10^{-7}	2.9656×10^{-7}	2.3898×10^{-7}	1.93×10^{-7}
2	$y(t-1)$	-0.1399	0.6097	0.6201	0.5417
3	$y(t-3)$	-0.3756	-0.0188	-0.01	-0.0292
4	$y(t-4)$	-0.935	-0.9352	-0.9384	-0.9548
5	$y(t-5)$	-0.0013	0.7857	0.802	0.7337
6	$y(t-3)^2$	-1.0056×10^4	-81.1288	43.185	-149.4653
7	$y(t-4)^2$	-1.352×10^4	394.7055	444.9489	429.4193
8	$y(t-5)^2$	-4.6026×10^3	-1.911×10^3	-1.5788×10^3	-1.7486×10^3

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9	$y(t-3)y(t-4)$	6.3501×10^3	469.3153	561.6396	513.0609
10	$y(t-3)y(t-5)$	8.2201×10^3	1.2876×10^3	1.3274×10^3	1.59×10^3

As shown in Fig 1, let the left-side as base line, the position parameter R is defined as dimensionless number corresponding to each of mass like' $1,2,\cdots,5$ '. The dynamic parametrical model for 5DOF nonlinear system was chosen to be

$$\begin{aligned} y(t) &= \theta_1(R)u(t-2)+\theta_2(R)y(t-1) \\ &+\cdots+\theta_9(R)y(t-1)y(t-2)+\theta_{10}(R)y(t-3)y(t-4) \end{aligned} \tag{29}$$

where the parameter $\theta_m(m=1,2,\cdots10)$ depends on the parameter R. Assume that parameter θ_m can be fitted using R, with a polynomial function below

$$\theta_m(R) = \beta_{m,0} + \beta_{m,1}R + \beta_{m,2}R^2 + \beta_{m,3}R^3 \tag{30}$$

The parameters $\beta_{m,n}(n=1,\cdots,3)$ can directly be estimated using the LS algorithm

$$\begin{cases} \theta_1(R)=1.1529\times10^{-6}-1.2316\times10^{-6}R+4.4911\times10^{-7}R^2-4.867\times10^{-8}R^3 \\ \theta_2(R)=-0.0819-0.135R+0.3079R^2-0.0558R^3 \\ \quad\vdots \\ \theta_{10}(R)=-1.7615\times10^4+2.5382\times10^4R-9.9343\times10^3R^2+1.1846\times10^3R^3 \end{cases} \tag{31}$$

The comparison between REFOR's predicted output and real system's output is presented in Fig.4.

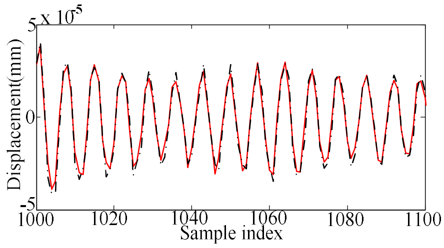
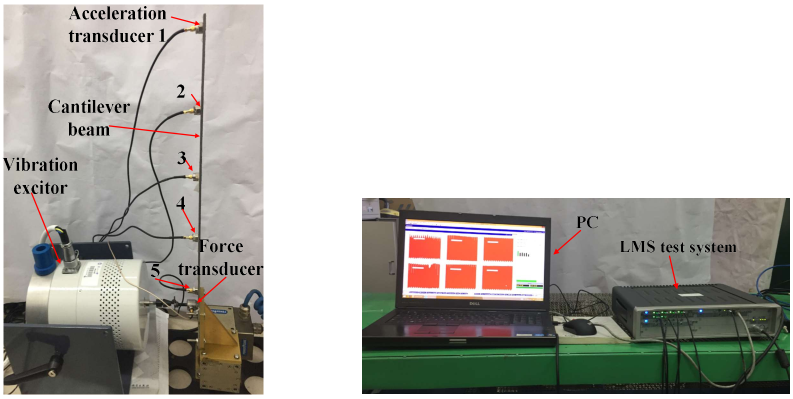


Fig.4 .A comparison among the REFOR's output and corresponding real output for system: —output of real system, ▪·▪REFOR output

Clearly, the REFOR's predicted output can provide an excellent representation for the test data sets.

2.4 Experimental validation

A cantilever beam is taken for experimental validation, with five acceleration transducers stick to it, each of them is 100mm apart. The force transducer is connected with vibration exciter by bolted joint, which transmits the excitation from power amplifier, and the other end stick to the bottom of cantilever beam. The input (force) and output (acceleration) was measured by LMS test system. The experimental set-up is shown in Fig.5.

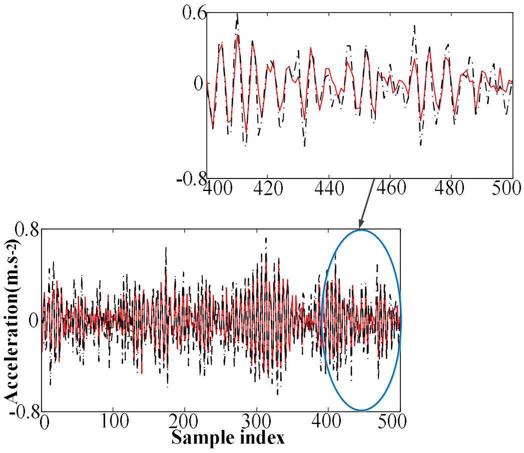


(a) Test rig of cantilever beam with transducers **(b)** LMS test system

Fig.5. Experimental setup

Four data sets, corresponding to transduce 1,3,4,5, with the same input, but different output, are used for model identification, and one data set, corresponding to transducer 2, was used to test the performance of the identified dynamic parametrical model.

The experimental scenario is confined to the industrial application, for example, structural damage detection of cantilever beam structure, and the input is a banded limited white noise signal [14].The comparison between REFOR's predicted output and corresponding measurements is presented in Fig.6.



corresponding real measurements: ——output of real system, ▬ ˈ ▪output of REFOR identified

Fig.6. A comparison between the REFOR's output and

3 Conclusions

EFOR selects model terms based on AERR criterion, which may missing some significant terms, and the underlying system can't be represented comprehensively. In a contrast, the new system identification algorithm REFOR includes the model terms

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as comprehensive as possible, which have been identified by corresponding different cases of NARX model, under the premise of a unified model structure. The new algorithm can provide a good prediction to the underlying system's output and avoid the defects of EFOR.

Furthermore, pre-specified threshold ρ directly affect the accuracy of model's prediction, the value of ρ will be discussed in the next paper.

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