# The Stability Study of Stick-slip Vibrations in Deep Hole Drilling

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**Abstract**: The deep hole drilling is widely used in gas production, but it often appears some serious accidents caused by stick-slip vibrations. Stick-slip vibration is essentially the development of the torsional vibration mainly due to the changing of cutting depth caused by the effect of cutting regeneration and the axial vibrations. Therefore, a linear stability analysis of the stick-slip vibrations, using a discrete model that takes into consideration the axial and torsional vibration modes, is described. Coupling between these two vibration modes takes place through a nonlinear delay cutting force caused by a bit-coal interaction.

The results show that the regeneration will greatly reduce the stability and the axial vibrations play a role to stabilize the torsional vibrations of the drilling system. The methods and results can be adopted for deep hole drilling stability prediction and provide reference for the dynamic optimization design.

**Keywords**: Stick-slip vibrations; Linear stability; Regenerative cutting

## 1 Introduction

The vibration of the drill string system cause serious damage to the coal mine gas extraction. And mine gas extraction is an important guarantee of safe and efficient mining of the coal mine. With the development of drilling technology, mine directional drilling technology is widely used in the extraction of coal seam gas. It uses drilling tool driven by the directional drilling to drill a pilot hole and extract gas in coal seam, see Fig.1.

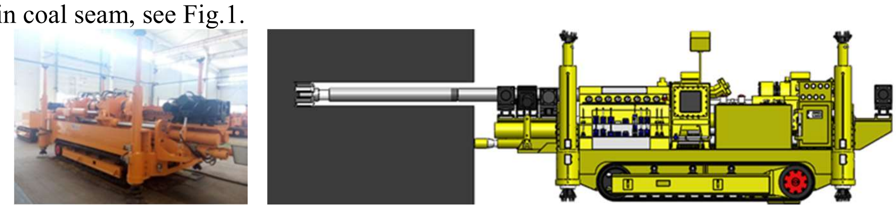

**Fig. 1.** The directional drilling and working sketch.

Vibrations can leads to a severe damage of the drill string including drill pipe and bit, causing sticking, plugging holes and other accidents. There are three forms of drill

*Corresponding Author (hli@mail.neu.edu.cn) string vibrations, including axial vibrations, torsional vibrations and lateral vibrations. The development of these vibrations can lead to bit bounce, stick–slip, and bit whirl, respectively [1]. In general, stick–slip is observed at low rotational velocity and high feed velocity of the bit [2], while bit whirl at high rotational velocity and low feed velocity [3]. Therefore, stick-slip vibration is the main form for deep hole drilling with low rotational velocity and high feed velocity. Stick-slip vibration is characterized by appearing alternately of stick phase and slip phase. In stick phase, the bit stops and the drill pipe continues to twist. In slip phase, the rotational velocity of the bit increases instantly because of the energy stored by the drill pipe in stick phase. This kind of phenomenon will periodically change stress and strain to accelerate the fatigue failure of drill string.

In recent years, many scholars analyze the stick-slip vibration of drill string system. Brett found the stick-slip vibrations existed only when bit contacts with coal through experiment [4]. Challamel confirmed the cutting process of bit is a kind of self-excited phenomenon and used the rock breaking mechanism to explain the basic principle of stick-slip vibration [5]. Nishimatsu analyzed the interaction between bit and coal through the single tooth cutting tool cutting experiment [6]. Richard studied the stability of equilibrium position of system, but he didn`t consider the effect of damping [7]. Germay used singular perturbation analysis to decouple axial and torsional dynamics in the mode [8], and then to conducted a detailed analysis of the axial limit cycle ensuring out of unstable drilling [9]. Besselink considered the axial damping in Germay`s model and analyzed the effect of axial periodic motion by semi-analytical method [10].

This paper uses a discrete model with axial and torsional damping that takes into consideration the axial and torsional vibrations to study the stability of the drilling system. The key points are the impact of regenerative cutting of the bit and axial vibrations on the stability of the drill string system. The dynamic model and a detailed stability analysis are described in section 2. Section 3 is the results. Finally, conclusions are drawn in section 4.

## 2 Dynamic model

### 2.1 Bit-coal interaction

The rotational velocity $\Omega_{rot}$ and feed velocity $V_0$ decided by the rig are the most important control parameters of deep hole drilling in coal seam. Thus, the dynamic characteristics and stability of the drill string system could be studied by analyzing the bit-coal interaction based on the control parameters.

The PDC bit (Polycrystalline Diamond Compact bit) is widely used in deep hole drilling in coal seam with good impact toughness and the ability to handle minor accident. According to the single blade cutting experiment of PDC bit [11], the action of such a bit consists of two independent processes, see Fig.2: (i) a cutting process taking place on the cutting face, (ii) a frictional contact process taking place on the interface between the wearflat and the coal. The force F on the bit results therefore from the superposition of two forces $F_c$ and $F_f$, acting on the cutting face and on the

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wearflat, respectively. The components of these two forces in a direction parallel (subscript s) and perpendicular (subscript n) to the cutter motion, i.e., $F_C = (F_{cs}, F_{cn})^T$, $F_f = (F_{fs}, F_{fn})^T$; can be expressed as:

$$F_{cs} = swd, \; F_{cn} = \zeta F_{cs}. \tag{1}$$

$$F_{fs} = \mu F_{fn}, \; F_{fn} = \sigma w\ell. \tag{2}$$

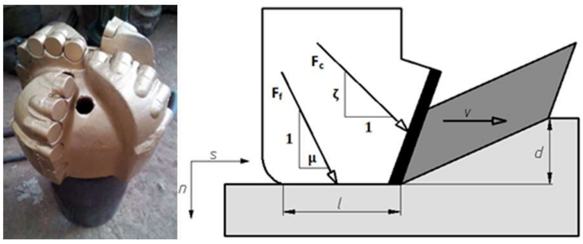


**Fig. 2.** The PDC bit and single blade cutting model.

Where d denoting the depth of cut and $\ell$ the length of the wearflat, w is a constant cutter width, s is the intrinsic specific energy, a parameter related to the coal strength under certain conditions, ζ is a number characterizing the orientation of the cutting force Fc, μ is a coefficient of friction and σ is the maximum contact pressure at the wearflat/coal interface.

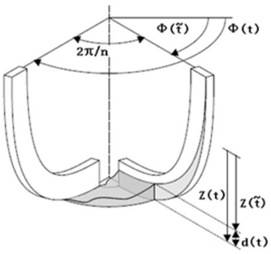


**Fig. 3.** The bottom-hole profile located between two successive blades of a PDC bit.

In this paper, a PDC bit has a set of N identical blades which spaced symmetrically around the axis of the bit is analyzed. And the weight and torque on the bit can be decomposed into two parts like the force on the bit: a cutting process (denoted by the subscript c) and a frictional contact process (denoted by the subscript f). As a result, these specific expressions can be written as follows:

$$T=T_c+T_f, \; W=W_c+W_f. \tag{3}$$

$$T_c = \frac{1}{2\zeta} a W_c, \; W_c = \zeta saNd. \tag{4}$$

$$T_f = \frac{1}{2} \mu a W_f, W_f = \sigma aN\ell. \tag{5}$$

Where a is the radius of the bit, d is the depth of cut per blade, see Fig.3, i.e., the axial displacement of bit from time $\tilde{t}$ to time t:

$$d(t) = Z(t) - Z(\tilde{t}). \tag{6}$$

where $\mathrm{Z}(t)$、$\Phi(t)$ are the axial and torsional position of the bit at time t, $\tilde{t} = \mathrm{t} - t_n$, $t_n$ is a state dependent delay time required for the bit to rotate an angle 2π/N to its current position at time t and determined by the following equation:

$$\Phi(t) - \Phi(\tilde{t}) = \frac{2\pi}{\mathrm{N}}. \tag{7}$$

There is a steady-state without vibrations of the bit and the space position （$Z_0, \Phi_0$） and the depth of cut are given by:

$$\Phi_0(t) = \Omega_{\mathrm{rot}} t,\ Z_0(t) = V_0 \mathrm{t}. \tag{8}$$

$$d_0 = \frac{2\pi}{N\Omega_{\mathrm{rot}}} V_0. \tag{9}$$

## 2.2 Dynamic equation

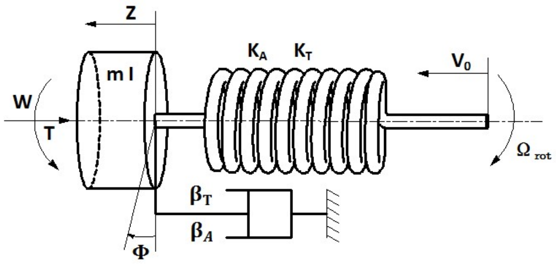


**Fig. 4.** The dynamic model of the drill string system.

A rotary drilling structure mainly consists of a rig and a drill string system (bit and drill pipe). The rig is considered to a boundary condition with a constant feed velocity $V_0$ and a constant rotational velocity $\Omega_{\mathrm{rot}}$. We consider a discrete model of the drill string system characterized by two degrees of freedom, Z andΦ, corresponding to the axial and angular position of the bit, respectively, and by six mechanical elements, namely a point mass m, a moment of inertia I, a spring of torsional stiffness $K_T$ and axial stiffness $K_A$, a damper of torsional damping $\beta_T$ and axial damping $\beta_A$, see Fig.4. The mass M and the moment of inertia I are taken to represent the bit, while the spring K and the damper $\beta$ are assumed to stand for the drill pipe.

In order to effectively analyze the vibrations of the drill string system, we write all dynamic equations in a moving coordinate system zϕ that moves along the Z-axis with feed velocity $V_0$ and rotates around the Z-axis with rotational velocity $\Omega_{rot}$:

$$z(t) = Z(t) - V_0 t, \phi(t) = \Phi(t) - \Omega_{\mathrm{rot}} t. \tag{10}$$

The dynamic equations can thus be written in the form:

$$\mathrm{m}\ddot{z} + \beta_A \dot{z} + k_A z = W_0 - \mathrm{W}[Z(t), \Phi(t)], \tag{11}$$

$$\mathrm{I}\ddot{\phi} + \beta_T \dot{\phi} + k_T \beta = T_0 - T[Z(t), \Phi(t)]. \tag{12}$$

Where $W_0$ and $T_0$ are the stationary weight and torque on the bit associated with the steady-state. Substitution of (10) into (7) yields the delay equation:

$$\frac{\phi(t) - \phi(\tilde{t})}{\Omega_{\mathrm{rot}}} + \left(t - \tilde{t} - \frac{2\pi}{N\Omega_{\mathrm{rot}}}\right) = 0. \tag{13}$$

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Substitution of (3)–(6), (8)–(10) (13) into (11) and (12) yields the perturbation dynamic equations:

$$\mathrm{m}\ddot{z} + \beta_A\dot{z} + k_A z = -\zeta \mathrm{saN}\left[z(t) - \mathrm{z}(\tilde{t}) - \mathrm{N}\frac{\phi(t)-\phi(\tilde{t})}{2\pi}d_0\right], \quad (14)$$

$$\mathrm{I}\ddot{\phi} + \beta_T\dot{\phi} + k_T\beta = -\frac{1}{2}\mathrm{sa}^2\mathrm{N}\left[z(t) - \mathrm{z}(\tilde{t}) - \mathrm{N}\frac{\phi(t)-\phi(\tilde{t})}{2\pi}d_0\right]. \quad (15)$$

### 2.3 Dimensionless dynamic equations

Stick-slip vibration is essentially the development of the torsional vibrations. So, the torsional natural frequency of the drill string is used to simplify the time scale τ. The steady-state depth of cut $d_0$ and the dimensionless cutting depth ψ are used to simplify the perturbed space position (z, $\phi$) of the bit, respectively.

To simplify the analysis the following parameters is introduced:

$$\tau = \omega_T t, \mathrm{u} = \frac{z}{d_0}, \varphi = \frac{\phi}{\psi}. \quad (16)$$

Where:

$$\omega_T = \sqrt{\frac{k_T}{I}}, \psi = \frac{sa^2 N d_0}{2k_T} = \frac{W_0 - W_f}{2\zeta k_T}. \quad (17)$$

Applying the above parameters, the perturbation dynamic equations (14)–(15) and delay equation (13) are given by:

$$\nu^2\ddot{u} + 2c_A\nu\dot{u} + ku = (1-k)\{-[u(\tau) - u(\tilde{\tau})] + \Psi[\varphi(\tau) - \varphi(\tilde{\tau})]\}, \quad (18)$$

$$\ddot{\varphi} + 2c_T\dot{\varphi} + \varphi = -[u(\tau) - u(\tilde{\tau})] + \Psi[\varphi(\tau) - \varphi(\tilde{\tau})], \quad (19)$$

$$\tilde{\tau} = \tau - \tau_n, \quad (20)$$

$$\tau_n = \Delta\tau\{1 - \Psi[\varphi(\tau) - \varphi(\tilde{\tau})]\}. \quad (21)$$

And the dimensionless parameters are given by:

$$\nu = \frac{\omega_T}{\omega_A}, \omega_A = \sqrt{\frac{\mathrm{k}_A + \zeta\mathrm{saN}}{\mathrm{m}}}, c_A = \frac{\beta_A}{2m\omega_A}, c_T = \frac{\beta_T}{2I\omega_T}, \kappa = \frac{\mathrm{k}_A}{\mathrm{k}_A + \zeta\varepsilon\mathrm{aN}},$$

$$\Omega = \frac{\Omega_{\mathrm{rot}}}{\omega_T}, \Psi = \frac{N\psi}{2\pi}, \Delta\tau = \frac{2\pi}{N\Omega}. \quad (22)$$

Parameter $\tau_n$ is the dimensionless delay time. Parameter $\omega_A$ is the natural frequency of the axial vibrations. Parameter $c_A$ and $c_T$ are the axial and torsional damping ratio, respectively. The parameter κ characterizes the axial stiffness of the drill string with respect to the coal strength. In the case of gas extraction, the following values of the dimensionless parameters can be considered: $\nu \sim 0.1, \mathrm{k} \sim 0.01$, $c_A \sim c_T \sim 0.05$，$\Psi \sim 10$.

### 2.4 Linear stability analysis

For the linear stability analysis, the perturbation dynamic equations (18)–(19) should be linearized. In this paper, the state dependent delay (SDD) time $\tau_n$ is considered to a constant delay (CD) time $\Delta\tau$ and the perturbation dynamic equations (18)–(19) could be transformed to a set of linear differential equations with the constant delay time.

To simplify the analysis, the linear differential equations were converted into the state vector equation through a state vector **x**:

$$\dot{\mathbf{x}}(\tau) - \mathrm{A}\mathbf{x}(\tau) - \mathrm{B}\mathbf{x}(\tau - \Delta\tau) = 0. \tag{23}$$

Where $\mathbf{x} = [x1, x2, x3, x4]^T = [\dot{u}, u, \dot{\varphi}, \varphi]^T$, A and B are the coefficient matrices for the current state vector and the delayed state vector, respectively. Details are given in Appendix A.

Then, the stability of the equilibrium position $\mathbf{x} = \mathbf{0}$ is analyzed and the characteristic equation of the state vector equation is given by:

$$\left|\lambda \mathrm{I} - \mathrm{A} - \mathrm{e}^{-\lambda\Delta\tau} B\right| = 0. \tag{24}$$

Where λ denotes an eigenvalue of the linearized system and the exponential term appears due to the time delay.

There are an infinite number of eigenvalues in the characteristic equation (24). The drill string system is stable only when all the eigenvalues have negative real part, otherwise the system is unstable. Pure imaginary eigenvalues (λ=iω,−iω) corresponding to a specific condition determine the stability boundaries that divide the stable and unstable regions. Assuming that the eigenvalue λ is equal to iω, the characteristic equation (24) could be converted into frequency equation:

$$\left[(-\omega^2 + 2\mathrm{i}\nu\omega c_A + \nu^2) - \nu^2(1-\mathrm{k})\left(1 - \mathrm{e}^{-\frac{\mathrm{i}\omega 2\pi}{\mathrm{N}\Omega}}\right)\right]\left[(-\omega^2 + 2\mathrm{i}\omega c_T + 1) - \psi\left(1 - \mathrm{e}^{-\frac{\mathrm{i}\omega 2\pi}{\mathrm{N}\Omega}}\right)\right] + \psi\nu^2(1-\mathrm{k})\left(1 - \mathrm{e}^{-\frac{\mathrm{i}\omega 2\pi}{\mathrm{N}\Omega}}\right)^2 = 0. \tag{25}$$

We can see this equation depends on seven dimensionless parameters, i.e., $\Omega, \Psi, \mathrm{N}, \nu, c_A, c_T$ and k. To get the stability boundaries of the drill string system, we can define a set of the parameters $\Omega, \Psi, \mathrm{N}, \nu, c_A, c_T$ and k, such that the imaginary part of the characteristic frequency ω in the frequency equation is equal to zero, i.e., $\mathrm{Im}(\omega) = 0$.

We use Euler's formula to simplify the exponential function $\mathrm{e}^{-\frac{\mathrm{i}\omega 2\pi}{\mathrm{N}\Omega}}$ and separate the frequency into real and imaginary parts lead to a set of algebraic equations consisting of polynomials. To manage the periodicity of the trigonometric function caused by $\mathrm{e}^{-\frac{\mathrm{i}\omega 2\pi}{\mathrm{N}\Omega}}$, we introduce a new variable parameters U defined by:

$$\mathrm{U} = \frac{\omega}{N\Omega}, \mathrm{U} \in \left(\mathrm{Max}\left(0, \ \mathrm{q} - \frac{3}{2}\right), \mathrm{q} - \frac{1}{2}\right]. \tag{26}$$

Where q=1, 2, 3… is a natural number marking different branches of $\mathrm{e}^{-\frac{\mathrm{i}\omega 2\pi}{\mathrm{N}\Omega}}$. For any given U the stability boundaries can be solved.

Stick-slip vibration is essentially the development of the torsional vibration. But, axial vibrations will change the cutting depth d corresponding to the equation (7) and affect the torsional vibration. To consider the impact of axial disturbance on the stability of drill string system, we first analyze the stability of axial vibrations and torsional vibration, respectively. Finally, the stability of coupled two degrees of freedom (DOF) drill string system is analyzed.


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# 3 Results

## 3.1 The stability of axial vibrations

In this case, the torsional stiffness is infinite and the parameter Ψ is equal to zero. So, the stability of axial drill string system can be characterized by the following five dimensionless parameters: κ、$c_A$、N and νΩ. We plot the stability charts for axial vibrations in the νΩ–κ plane with fixed parameters N (N=4) and $c_A$ ($c_A = 0.05{,}0.25$).

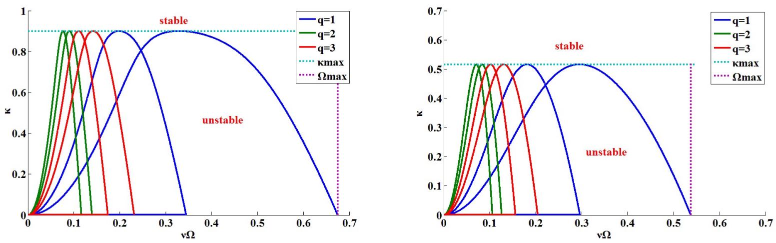


**Fig. 5.** Stability charts of axial vibrations:（a）$c_A$=0.05；（b）$c_A$=0.25.

The drill string system is stable in the upper part of the curve and unstable in the lower part of the curve, see Fig.5. The stability of the drilling system is increase with rising values of κ, a parameter contains information about the axial stiffness of the drill string and the strength of the coal. Note that the parameter κ is less than one (κ< 1) and increases as the axial stiffness of the drill pipe increases or the strength of the coal decreases. The higher axial damping ratio $c_A$ could enhance the stability of the drill string system by comparison with Fig.5 a, b.

The lobes of axial stability chart represent the regenerative effect caused by the cutting action of the bit. In fact, each blade of the vibrating bit leaves undulating rock surface, which, in turn, being subsequently cut by the following blade, further excites the vibrations of the bit. The parameter q=1,2,3,… corresponds to the number of lobes.

The impact of the cutting action of the bit on the axial drill string system is similar to a spring. Assuming that torsional angle Φ and delayed displacement z(t̃) is equal to zero in the axial dynamic equation (14). The axial stiffness of the cutting process is equal to ζsaN (ζsaN > 0). The total axial stiffness of the drill string system is $k_A +$ ζsaN ($k_A +$ ζsaN $> k_A > 0$). Thus, the cutting process of the bit will increase the total axial stiffness and stability of the drill string system. The stability boundary of the axial drill string system is κ = 1 without regenerative cutting action of the bit.

To consider the regenerative cutting action, the axial regeneration term $[\xi(\tau) - \xi(\tilde{\tau})]$ will increase vibration and decrease stability of the drill string system, see Fig.5. And the whole unstable region is limited to following region:

$$\kappa \le \kappa_{\max} = \frac{{c_A}^{-2} - 2\sqrt{{c_A}^{-2} - 1}}{{c_A}^{-2}}, \Omega \le \Omega_{\max} = \frac{1}{\nu N}\frac{2\pi\sqrt{2 - 4{c_A}^2}}{2\pi - \arccos\left(4{c_A}^2 - 1\right)}. \tag{27}$$

### 3.2 The stability of torsional vibrations

In this case, the axial stiffness is infinite and the parameter κ is equal to one, ( $\kappa = 1$). So, the stability of torsional drill string system can be characterized by the following four dimensionless parameters: $\Psi$,$c_T$,N and $\Omega$. We plot the stability charts for torsional vibrations in the Ω–Ψ plane with fixed parameters N (N=4) and $c_T$ ($c_T = 0.1,0.25$), see Fig.6.

The drill string system is stable in the lower part of the curve and unstable in the upper part of the curve. The stability region of torsional system is bigger with smaller parameter Ψ, the dimensionless cutting depth, because the cutting force will increase with increasing cutting depth, which caused by the bit-coal interaction. The torsional ratio $c_T$ can improve the stability of the torsional drilling system which can be seen from the Fig.6 a, b.

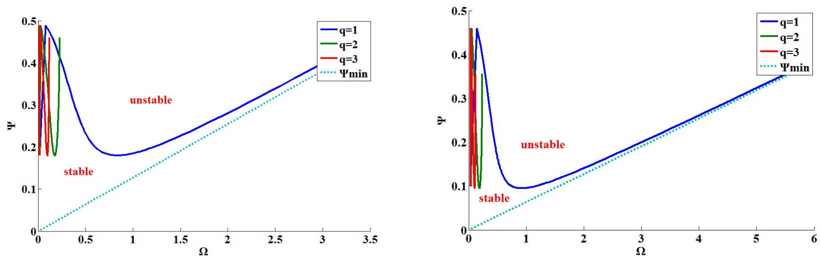


**Fig. 6.** Stability charts of torsional vibrations: (a)$c_T$=0.1; (b) $c_T$=0.25.

Assuming that axial displacement z and delayed angle $\Phi(\tilde{t})$ is equal to zero in the torsional dynamic equation (15). The torsional stiffness of the cutting process is equal to $-(1/4\pi)sa^2N^2d_0$ ($-\left(\frac{1}{4\pi}\right)sa^2N^2d_0 < 0$). The total torsional stiffness of the torsional drill string system is $k_T - \left(\frac{1}{4\pi}\right)sa^2N^2d_0$. The system will be very unstable if the total torsional stiffness is less than zero ($k_T - (1/4\pi)sa^2N^2d_0 < 0$). The criterion $k_T - (1/4\pi)sa^2N^2d_0 > 0$ is equivalent to$\Psi < 1$. Thus, the stability boundary of the torsional drill string system is $\Psi = 1$ without regenerative cutting action of the bit.

Considering the torsional regenerative cutting action $[\varphi(\tau) - \varphi(\tilde{\tau})]$ of the bit, the stable region reduced to the area in the Fig.6 and the whole instability region is located above the following asymptote:

$$\Psi_{\min} = \frac{2\Omega c_T}{(2\pi/N)}. \tag{28}$$

### 3.3 The stability of coupled axial-torsional vibrations

Fig.7-9 are the stability charts in the Ψ-Ω plane in the case of coupled axial-torsional vibrations. Comparing these charts with the pure torsional vibration`s, we will see the stability of coupled vibrations is enhanced.

In Fig.7, the stability of drill string system is gradually increases with the decreasing of parameter κ. On the other hand, the stability is gradually increases with the

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increasing of axial damping ratio $c_A$ which is different from pure axial vibrations , see Fig.8. The system has a better stability with smaller parameter v, the ratio of the torsional to axial frequencies of vibrations, see Fig.9. So, we can see the axial vibrations of drilling system plays a role to stabilize the torsional ones. This is because the fast axial vibrations can rapidly reduce any changes in the depth of cut caused by slow torsional vibrations.

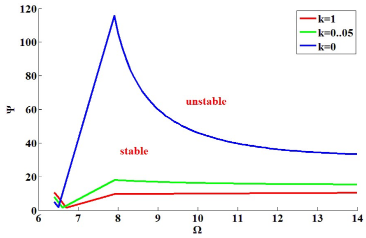


**Fig. 7.** Stability charts of axial–torsional vibrations, effect of axial stiffness κ: N=4,v=0.1,$c_A$=0.05,$c_T = 0.05$.

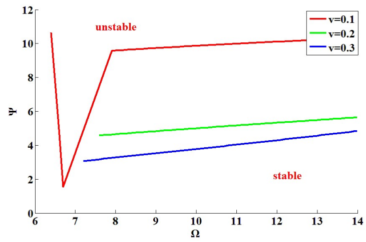


**Fig. 8.** Stability charts of axial–torsional vibrations, effect of axial stiffness v: N=4,k=0,$c_A$=0.05,$c_T = 0.05$.

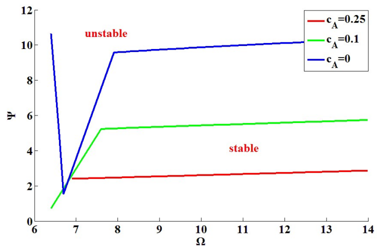


**Fig. 9.** Stability charts of axial–torsional vibrations, effect of axial stiffness $c_A$:

N=4,k=0,v=0.01,$c_T = 0.05$.

## 4 Conclusions

Stick-slip vibration is a special form of the torsional vibrations and caused by bit-coal interaction. To analyze this vibration of drill string system, its 2 DOF torsional mass-spring-dashpot model is established. For deep hole drilling in coal seam, the changing cutting depth caused by the regenerative cutting action of the bit will change the cutting force and lead to strong stick-slip vibrations, one kind of self-excited vibrations. Thus, dynamic equation of the drill string system is a state dependent delay equation.

To avoid the stick-slip vibrations and analyze the effect of axial vibrations on torsional vibrations, a linear stability analysis of coupled axial–torsional vibrations has been carried out. We analyzed the stability of the axial vibrations, torsional vibrations and coupled axial-torsional vibrations of the drill string system with a constant delay which is an approximation of state dependent delay, respectively.

The results show that the regeneration cutting of the bit will greatly reduce the stability of the drill string system and the axial vibrations play a role to stabilize the torsional vibrations by comparing the stability of axial vibrations, torsional vibrations and coupled axial–torsional vibrations. Because the fast axial vibrations can rapidly reduce any changes in the depth of cut and enhance the stability. The methods and results can be adopted for deep hole drilling stability prediction and provide reference for the dynamic optimization design.

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## Appendix A. Coefficient matrices A and B in (22)

$$A = \begin{bmatrix} -\nu^{-1}Q_A^{-1} & -\nu^{-2} & 0 & (1-k)\nu^{-2}\Psi \\ 1 & 0 & 0 & 0 \\ 0 & -1 & -Q_T^{-1} & \Psi - 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{A1}$$

$$B = \begin{bmatrix} 0 & (1-k)\nu^{-2} & 0 & -(1-k)\nu^{-2}\Psi \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\Psi \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A2}$$

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