Research on optimal replacement strategy for single equipment based on economic life

Qingyu Zhu[1*], Qing Wang[1], Xiaowei Gu[1], Hui Chen[2]

1. School of Resources and Civil Engineering, Northeastern University, Shenyang 110819, China
2. School of Mechanical Engineering, Dalian University of Technology, Dalian 116024, China

Abstract: An economic life model of single equipment is built, the static economic life and the dynamic economic life of the equipment are calculated based on the cash flow for expenditure and comprehensive production efficiency. Based on the basic principle of dynamic programming, the replacement strategy optimization model of single equipment is established, and the optimal replacement strategies of the equipment under six different finite time domain conditions are analyzed and compared. The results show that the service lives of the equipment in all six optimal replacement strategies are less than their dynamic economic lives. Thus, the dynamic economic life can't be used as the replacement standard of equipment in solving the problem of optimal replacement strategy of equipment.

Keywords: Single equipment; Economic life; Equipment replacement; Dynamic programming; Optimal strategy

1 Introduction

Equipment is an important material and technical foundation of modern industrial production, and also an important factor affecting the economic and technical indexes of enterprises and national economy. Equipment inevitably deteriorates with age, resulting in higher operating and maintenance (O&M) costs and lower utilization rate of equipment, which have a great influence on enterprises' comprehensive economic benefits. In recent years, people have been paying more and more attention to optimal replacement strategy of equipment.

In 1955, Bellman[1] firstly proposed dynamic programming formulation for the equipment replacement problem. In 1975, Wagner[2] presented a new dynamic programming model that relaxed the assumption of repeatability, however, it required a finite horizon time due to the solution technique restriction. Bean et al.[3] established the dynamic decision-making model to solve equipment replacement problems involving infinite horizon time. In early 1980s, Professor Fu from Tsinghua University took the lead in carrying out researches on equipment replacement problem in China.

* Corresponding author: Qingyu Zhu (neuzqy@foxmail.com)

Through numerous study cases and theoretical analyses, Fu theorized equipment replacement, put forward the economic life analysis model and formed a complete set of method system for equipment replacement problem. Murphy and Hartman[4] described equipment replacement as an integer-knapsack problem with a nonlinear objective function, and proposed another dynamic-programming formulation for the finite-horizon equipment replacement problem. However, few papers attempted to establish links between the optimal replacement and technological change (TC) in previous researches. If a new device with higher efficiency and better economic results during the service life of the equipment, two schemes of keeping using old device and purchasing new device should be compared in order to figure out which scheme is more economical. Equipment technological change was taken into account by Sharp[5] and Yuri Yatsenko[6], and they discussed links between the optimal replacement and TC under finite horizon. There are many researches on incorporating the economic life and TC into replacement strategy optimization [7-9].

This paper is organized as follows. We briefly review the concept of economic life in Section 1, and establish an economic life model of the equipment, including static and dynamic model. Section 2 presents a dynamic programming model for single-equipment optimal replacement problem under finite horizon. Section 3 examines six optimal replacement strategies with different finite horizon, obtaining a general rule that equipment service life is less than its dynamic economic life, and corresponding analysis is discussed. The final section presents conclusions for the research.

2 Economic life

Production efficiency and production cost of equipment change with the increase of service age, called “economic life”. Based on single-equipment expenditure cash flow and annual production efficiency, the economic life model is built below.

From an investment perspective, the longer equipment serves, the more work is done, and the lower capital cost allocated into unit work is. For operating cost, higher annual operating cost and lower amount of work with longer working life may lead to higher operating cost allocated into unit work. Considering both factors above, generally, cost allocated into unit work presents a down and up trend with a U-shaped curve. So, there is a working life where cost allocated into unit work is the minimum, (i.e., economic life). The equipment will remain a residual value when it is scrapped, known as salvage, equipment salvage with an age of n years is expressed by $S(n)$.

If the equipment is used for n periods, the average total cost allocated to unit quantity of work done can be expressed as follows.

$$c(n) = \frac{CC + \sum_{t=0}^{n-1} OC(t) - S(n)}{\sum_{t=0}^{n-1} W(t)} \tag{1}$$

Where：

ISSN 2572-4975 (Print), 2572-4991 (Online)

$c(n)$: total cost allocated to unit quantity of work if the equipment is used for n periods;

CC : capital cost of the equipment;

$OC(t)$: annual operating cost of the t-year-old equipment;

$W(t)$: annual production efficiency of the t-year-old equipment.

Economic life is a period during which the total cost of unit work done is minimum, mathematically, economic life of the equipment is service life *n**, which minimizes the value of $c(n)$.

$$n^* = \min_{1\le n\le N}\{c(n)\} = \min_{1\le n\le N}\left\{\frac{CC+\sum_{t=0}^{n-1}OC(t)-S(n)}{\sum_{t=0}^{n-1}W(t)}\right\} \tag{2}$$

where N is natural life of the equipment.

Time value of the money is ignored in the established economic life model of equipment, so it is called static economic life. The dynamic economic indexes would usually be adopted in evaluation of economic benefits for any project, in other words, each cost needs to be discounted. During dynamic evaluation, we need to correctly handle the change of the equipment operating cost with age. This change does not refer to the change with different factors like age or equipment operating conditions, but the difference between the current annual operating costs and the annual operating costs of the equipment after several years under the same operating conditions, i.e., rise and down in price. In the normal economic state, operating cost of the equipment rises up in general. Therefore, we should predict a real cost escalation rate throughout the calculation period. Dynamic economic life is denoted as n_d^* to distinguish from static economic life *n**, if taking a real cost escalation rate into consideration, dynamic economic life formula can be written as:

$$n_d^* = \min_{1\le n\le N}\{c_d(n)\} = \min_{1\le n\le N}\left\{\frac{CC+\sum_{t=0}^{n-1}\frac{OC(t)(1+r)^{t+1}}{(1+d)^{t+1}}-\frac{S(n)(1+r)^{n+1}}{(1+d)^{n}}}{\sum_{t=0}^{n-1}W(t)}\right\} \tag{3}$$

where *d* is discounted rate; *r* is a real cost escalation rate.

Fig. 1 describes changes of cost and discounted cost per unit of work of the equipment with service life.

Initial investment *CC* of a new device is \$ 2.732 million, its production efficiency *W*(0) is 3.80 $Mt\cdot a^{-1}$, and its operating cost *OC*(0) is 247,830 $\$\cdot a^{-1}$. The natural life N of the equipment is 20 years, discounted rate d and real cost escalation rate r are respectively taken 6.8% and 1.5%. Here is a concise and accurate calculation method for salvage, X=Y(1-0.55ZS), where X is theoretical salvage of the equipment, Y is investment capital of the equipment, Z is depreciation rate(usually 10%~16%, it depends), S is service life. Further, simplify salvage calculation via combining accelerated depreciation methods, in which basic depreciation expense in first year is equal to half of original value of fixed assets, and then the other half value is fully depreciated in 5~7 years at the depreciation rate of 15%.

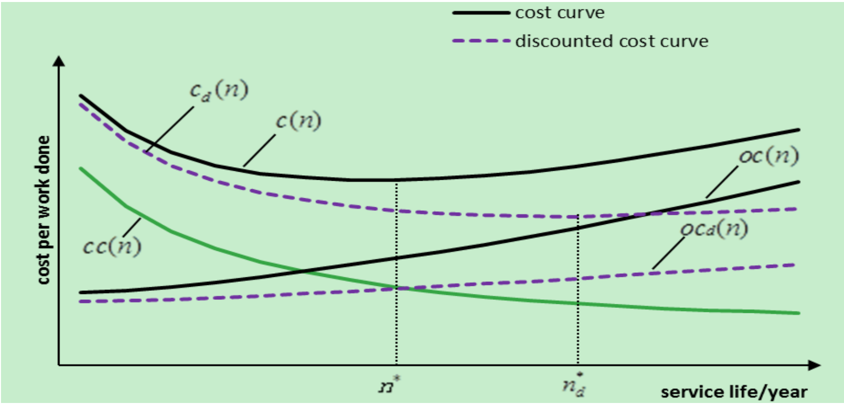

Fig. 1. Change curve of cost and discounted cost per unit of work with equipment service life

Table 1 shows annual production efficiency function *W(t)*, annual operating cost function $OC(t)$, and salvage function $S(n)$ for a certain type of equipment with a natural life of 20 years. The calculated average cost and discounted cost per unit of work done $c(n)$ and $c_d(n)$ are listed in Table 1 as well. According to definition, economic life is the life of equipment during which the average cost per unit of work done $c(n)$ or the average discounted cost per unit of work done $c_d(n)$ is minimum, as shown in Table 1, static economic life of the equipment is calculated to be 8 years, likewise, dynamic economic life of the equipment is calculated to be 15 years.

Table 1. Economic and technological parameters and economic life result for certain equipment

Service life n /a	Operating efficiency $W(t)$ /10^4t·a^{-1}	Operating cost $OC(t)$ /10^4\$·a^{-1}	Salvage $S(n)$ /10^4\$	Cost per work $c(n)$ /\$·t^{-1}	Discounted cost per work $c_d(n)$ /\$·t^{-1}	Working life n /a
0	380	24.783				
1	375	24.783	136.667	0.4247	0.4393	1
2	370	25.217	114.058	0.2764	0.2862	2
3	360	26.087	102.754	0.2179	0.2245	3
4	350	27.971	91.449	0.1902	0.1935	4
5	340	31.014	80.290	0.1753	0.1752	5
6	330	34.928	68.986	0.1674	0.1636	6
7	320	38.986	57.681	0.1638	0.1559	7
8	310	42.754	46.377	**0.1630**	0.1508	**8**
9	300	45.652	35.217	0.1641	0.1473	9
10	288	49.275	23.913	0.1663	0.1447	10
11	275	53.043	12.609	0.1697	0.1430	11
12	260	56.522	9	0.1741	0.1420	12
13	243	59.855	0	0.1771	0.1403	13
14	225	63.623	0	0.1808	0.1393	14
15	205	66.812	0	0.1857	**0.1389**	**15**
16	187	69.130	0	0.1915	0.1391	16
17	170	70.725	0	0.1980	0.1397	17
18	155	71.449	0	0.2050	0.1406	18
19	135	71.739	0	0.2123	0.1416	19
20	110	71.884	0	0.2200	0.1428	20

ISSN 2572-4975 (Print), 2572-4991 (Online)

3 Dynamic programming model

For a given finite horizon, optimal replacement problem of single equipment can be expressed as: determine when to replace the equipment in order to make the discounted cost allocated into per unit of work done in the whole period be minimum. Single-equipment replacement issue is a multi-stage decision problem which can be solved by a dynamic programming method.

Firstly, set up a dynamic programming model for equipment replacement before solving a multi-stage decision problem via a dynamic programming method. The following concepts are often used [10]:

(1) stage; (2) state; (3) decision and strategy; (4) state transfer equation; (5) target function.

Combined with replacement problem, the above concepts are illustrated in detail.

(1) Break down the process of the given problem into several related stages so as to find the solution of each stage in order. As for replacement problem, stage variables are defined as time: the initial stage is stage 0, corresponding to beginning of a finite horizon; each stage corresponds to the end of each year, in other words, stage 1 represents the end of the first year, stage 2 represents the end of the second year, and so on, final stage (stage M) is the end of a finite horizon with M years.

(2) Different stage have different states denoted by state variables. In this problem, state variables are defined as equipment service life. $S_{i,\,j}$ represents the jth state in the ith stage. As for different states in the ith stage, they signify different years the equipments have been serving when production proceeds to the end of ith year.

(3) Decision-making is a decision that we can make in a certain state of a certain stage to confirm a state in next stage. After serving for one year, the equipment can be kept or replaced ("K" and "R" in Fig. 2 represent "keep" and "replace", respectively): if keep using the equipment, move along the arrow marked with "K" to a state in next stage, with one year added to the service life of the equipment; if replace the equipment, move along the arrow marked with "R" to a state in next stage, the equipment being new with service year being 0.

Expenditure caused by a state transition is denoted by $c_{i,\,j}(i\text{-}1,\,k)$. If keep using the equipment, expenditure of that year only includes operating cost $OC(k)$; and if replace the equipment, expenditure of that year is equal to operating cost $OC(k)$ plus investment CC of purchasing a new piece of equipment, subtracting equipment salvage $S(k+1)$. So, for i = 1, 2, ... , M-1

$$c_{i,j}(i-1,k)=\begin{cases} OC(k)(1+r)^i & \textbf{Keep} \\ \left[OC(k)+CC-S(k+1)\right](1+r)^i & \textbf{Replace} \end{cases} \tag{4}$$

The strategy is defined as a sequence of "keep" or "replace" decisions in each stage over the horizon M. From the initial state $S_{0,\,0}$, stage by stage, the replacement path formed by moving along the selected route to a certain state of the last stage $S_{M,\,j}$ （j=1，2，...，M）may be an optimal replacement policy.

(4) The state of a certain stage is often determined by the state and the decision of

previous stage. If the states and decisions in the *k*th stage are given, states in (k+1)th stage are completely determined. Each arrow in Fig.2 is called a **state transition**. When $S_{i,\ j}$ is transferred from the previous state $S_{i\text{-}1,\ k}$, in the process of this state transition, an equipment with the service year being k was in use during the *i*th stage, and work done $w_{i,\ j}(i\text{-}1,\ k)$ can be written：

$$w_{i,j}(i-1,k) = W(k) \tag{5}$$

where $W(k)$ is annual operation efficiency of the *k*-year-old equipment.

When $S_{i,\ j}$ is transferred from the previous state $S_{i\text{-}1,\ k}$, the cumulative quantity of work $W_{i,\ j}(i\text{-}1,\ k)$ from the initial state $S_{0,\ 0}$ to state $S_{i,\ j}$ can be obtained as:

$$W_{i,j}(i-1,k) = W_{i-1,k} + w_{i,j}(i-1,k) \tag{6}$$

where $W_{i\text{-}1,\ k}$ is the cumulative work from the initial state $S_{0,\ 0}$ to state $S_{i\text{-}1,\ k}$ along an optimal path, and $W_{i\text{-}1,\ k}$ has been obtained while evaluating state $S_{i\text{-}1,\ k}$ in stag i-1.

By this path, cumulative expenditure present value and discounted cost per work done from the initial state $S_{0,\ 0}$ to state $S_{i,\ j}$, is denoted by $PC_{i,\ j}(i\text{-}1,\ k)$ and $pc_{i,\ j}(i\text{-}1,\ k)$, we can obtain:

$$pc_{i,j}(i-1,k) = \frac{PC_{i,j}(i-1,k)}{W_{i,j}(i-1,k)} = \frac{PC_{i-1,k} + \dfrac{c_{i,j}(i-1,k)}{(1+d)^i}}{W_{i,j}(i-1,k)} \tag{7}$$

(5) Target function is used to measure whether the selected strategy is good or not. As previously mentioned, the so-called "optimum" refers to the minimum discounted cost allocated into per work done over the whole finite horizon. Obviously, when states transfer from different previous states (take different values of k), the equipment finishes various amounts of work with different cumulative expenditure present value, thus discounted cost per work done of states $S_{i,\ j}$ is different. Hence, there is an optimal previous state among m previous states of $S_{i,\ j}$, from which to state $S_{i,\ j}$, we can gain the minimum $\boldsymbol{pc_{i,\ j}(i\text{-}1,\ k)}$, and this is called **optimal state transition** or **optimum decision**.

$$pc_{i,j} = \min_{k \in m}\left\{ pc_{i,j}(i-1,k) \right\} \tag{8}$$

After obtaining the minimum $\boldsymbol{pc_{i,\ j}(i\text{-}1,\ k)}$ over the whole finite horizon, get all decisions of each year via forward-recurrence method, through which a path with the minimum discounted cost per work done is acquired, and it is the path that is defined as an optimal strategy.

Fig. 2 depicts the dynamic programming path of the single-equipment replacement strategy with a certain finite horizon.

4 Case study

In this section, a certain type of mining shovel is taken as an example to analyze optimal replacement strategy of single equipment with different mine residual lifetime, by applying dynamic programming model established above. Since the static economic life and dynamic economic life of mining shovel are calculated to be 8 years and 15

ISSN 2572-4975 (Print), 2572-4991 (Online)

years, optimal replacement strategies of the mining shovel with different mine residual lifetime of 12 years, 15 years, 20 years, 30 years, 35 years, and 45 years are researched, respectively.

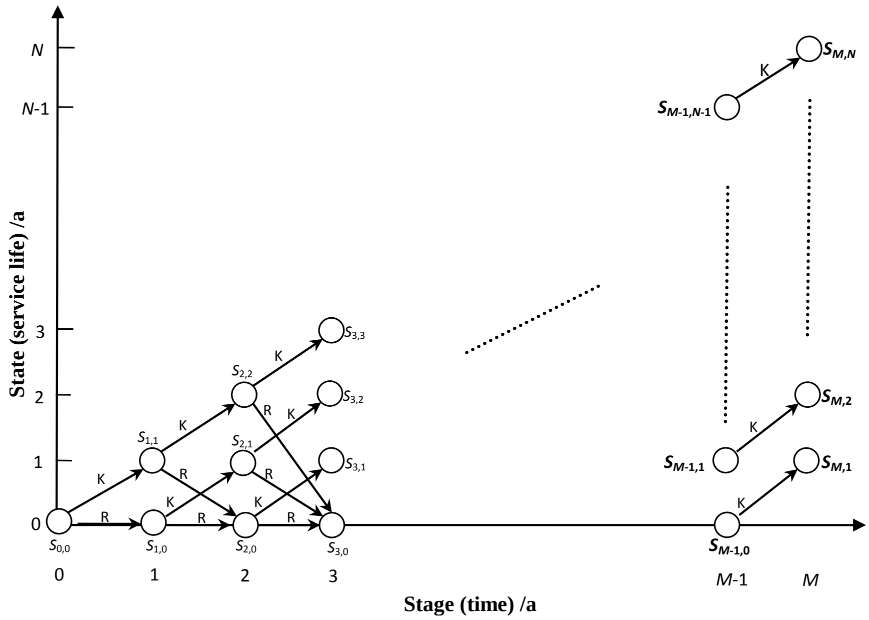

Fig. 2. Dynamic programming graph of single-equipment replacement strategy with a certain finite horizon

When mine residual lifetime is 12 years, we figure out the best service life of the shovel to be 12 years via dynamic programming, that is, no replacement till the end of mine will be a best choice.

When residual life of mine is equal to dynamic economic life of the shovel (15 years), according to general understanding of equipment replacement, the shovel happens to reach its dynamic economic life at the end of mine, and keeping the shovel in service until the end of mine seems to be the best choice. To verify this point, we change the residual life of mine into 15 years without varying other conditions, to optimize equipment replacement path by dynamic programming. It turns out that the shovel's best service life is still not its dynamic economic life but 7 or 8 years; the discounted cost per ton is \$ 0.1287 for the "optimal replacement strategy" and \$ 0.1389 for the "serving for 15 years", 7.93% higher than the "optimal replacement strategy".

When the residual life of mine is equal to an integral multiple of dynamic economic life of the shovel, we change mine residual life into 45 years, and launch a new shovel at time zero, is this the optimal replacement strategy that we replace the old shovel at the end the 15^{th} year and 30^{th} year? For M is equal to 45 years, as is shown by the optimization results, the shovel's best service life is still not its dynamic economic life but 8, 9 or 10 years (it is a coincidence not a rule that optimal service life is equal to static economic life); the discounted cost per ton is $ 0.07186 for the "optimal replacement strategy" and $ 0.07372 for the "serving for 15 years", 2.61% higher than the "optimal replacement strategy".

Consider a common situation, and we change mine residual life into 35 years, the optimization results show that the shovel's best service life is still not its dynamic economic life but 8, 9 or 10 years as well; the discounted cost per ton is $ 0.08548 for the "optimal replacement strategy" and $ 0.09128 for the "serving for 15 years", 6.78% higher than the "optimal replacement strategy".

Optimal equipment replacement strategies with different residual life of mine being 12 years, 15 years, 35 years, and 45 years are studied respectively. In these cases, the best service lives of the shovel are all less than its dynamic life. Fig.3 depicts six optimal replacement strategies (including four ones above) with different residual life of mine.

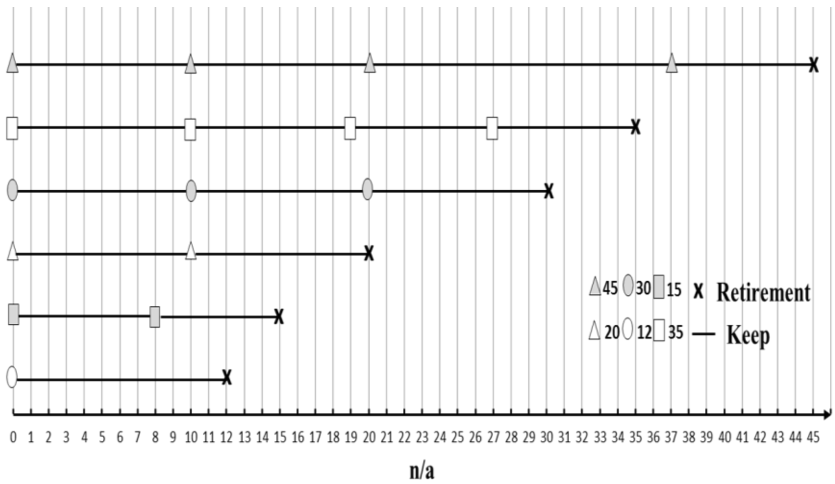

Fig. 3. Shovel's optimal replacement strategy under different residual life of mine

As is depicted in Fig.3, the best service lives of the shovel in these six cases are all less than its dynamic life. In fact, optimized results are the same for a mine with residual life less than 45 years. When residual life of mine is equal to an integral multiple of dynamic economic life of the equipment (particularly the same), it sounds even contradictory and seems hard to understand that dynamic economic life is not the optimal time to replace equipment. Dynamic economic life is a period during which the average discounted cost per work done is the minimum, and the cost per work done will increase with either longer service life or shorter service life. The optimization goal of dynamic programming is to minimize the discounted cost per work done, and why we could get less cost without replacing a piece of equipment in its dynamic economic life? The origin of which lies in:

When identifying the dynamic economic life and the optimal strategy of the equipment, we have actually compared different objects: for the evaluation of dynamic economic life, discounted costs per work done of **the same equipment** with differ-

 ISSN 2572-4975 (Print), 2572-4991 (Online)

ent service lives are compared, dynamic economic life of the equipment is a more economical service life than that of **the same equipment** serving longer or shorter; for the evaluation of optimal replacement strategy, two different discounted costs per work done are compared: ① **discounted cost per work done of the equipment over n years for continuous service; ② discounted cost per work done of the equipment over n-k years, plus discounted cost per work done of new equipment over k years.** For example, when residual life of mine is equal to dynamic economic life of the shovel (15 years), which is economical between using the shovel for 7 years and for 15 years？What we are comparing here are discounted cost per work done of the shovel with service lives of 7 years and 15 years, and this is the evaluation of dynamic economic life. However, in the process of evaluating optimal replacement strategy, we compare discounted cost per work done of the shovel with service lives of 15 years with that of two shovels (a shovel serves for 7 years, and a new shovel serves for another 8 years, totally 15 years).

5 Conclusions

(1) Some researches on equipment configuration and replacement took dynamic economic life of the equipment as a criterion for replacement. It turns out that we can’t get an optimal equipment configuration and replacement strategy by this way.

(2) It remains to be proven or verified **whether it is a general rule that the best service life of the equipment is less than its dynamic life.**

(3) In the paper, we merely conduct a research on equipment replacement problem without taking TC factor into consideration. Equipment replacement problem under multi-restriction (including TC problem, etc.) will be the direction of future research.

References

1. Bellman Richard, Equipment Replacement Policy, ***Journal of the Society for Industrial & Applied Mathematics*** 3 (1955) 133-136.
2. Wagner, Harvey M, ***Principles of operations research:with applications to managerial decisions /-2nd ed***, Prentice-Hall, (1975).
3. James C Bean, Jack R Lohmann, Robert L Smith, A Dynamic Infinite Horizon Replacement Economy Decision Model, ***The Engineering Economist*** 30 (1984) 99-120.
4. Murphy A, Finite-horizon equipment replacement analysis, ***IIE Transactions***, 38 (2006) 409-419.
5. Sharp G, Replacement under ongoing technological progress, ***IIE Transactions*** 36 (2004) 497-508.
6. Yuri Yatsenko, Natali Hritonenko, Technological breakthroughs and asset replacement, ***The Engineering Economist*** 54 (2009) 81-100.
7. Hartman, J C, Tan C H, Equipment replacement analysis: a literature review and directions for future research, ***The Engineering Economist*** 59 (2014) 136-153.
8. G Bethuyne, Optimal replacement under variable intensity of utilization and technological progress, ***The Engineering Economist*** 43 (1997) 85-105.

9. Cheevaprawatdomrong T, Smith R L, A paradox in equipment replacement under technological improvement, ***Operations Research Letters*** 31 (2003) 77-82.
10. Frederick S Hillier, Gerald J Lieberman, ***Introduction to operations research***, Tsinghua University Press, (2006).

ISSN 2572-4975 (Print), 2572-4991 (Online)