Synthesis Method for Improving the Structure Accuracy of Series Robot Based on SDT

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Abstract: In this paper, we focus on the need for improving the structure accuracy of series robot. The tolerance zone of a feature are described based on the SDT (Small displacement torsor) theory. The 3D tolerance model and assembly constraints of the robot joint group are established. And then, they are input into the kinematic error model of the robot as an error operator. By analyzing the sensitivity of the key error sources that affect the pose of the end-effector of the robot, the structural accuracy of the robot body is analyzed and optimized. By this method, the basic accuracy of the robot can be improved without increasing the manufacturing cost. And the efficiency of the kinematic calibration of the robot can be improved partly.

Keywords: Series robot; structural accuracy; 3D tolerance model; Small displacement torsor; sensitivity analysis

1 Introduction

Off-line programming requires that the actual structural parameters of the robot should be equal to the theoretical parameters given in the design. The structural errors of the links and the joints will result in greater deviations between the actual pose and the theoretical ones. Although the robot can be calibrated to eliminate the deviations, the calibration must be based on the basis accuracy of the robot body. The new GPS (Product Geometric Specification) standard system is built on the more clear tolerance semantics and measurement standards. With the applications for SDT (small displacement torsor) theory to tolerance analysis and synthesis, it is possible to prediction structure design error for the robot body in design. It is an economically feasible method to improve the kinematic accuracy of the robot by optimizing the tolerance of the functional features of the robot without increasing the cost

Most scholars are committed to the study for industrial robot error analysis and precision optimization. By using the D-H method to establish the kinematic relationship between the joint parameters and the end-effector‘s pose, the differential Jacobian matrix is the most widely used to build error models of the robot[1-3]. Some reports treat the error is treated as a small displacement vector to construct the pose error equation for the robot[4]. With the development of computer aided tolerance design technology,

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some optimization algorithms have applied to the design and distribution of improving the accuracy of the mechanism, which be used to obtain the tolerance instead by experience. A general kinematic modeling method for robot structural error analysis is proposed, which can estimate the pose error of the robot or the tolerances given at the designing stage[5-8]. The genetic algorithm is used to calculate the single error (straightness error, sphere error, roundness error, etc.), and the target error is calibrated reasonably[9]. The parallel tolerances were optimized using simulated annealing[10].

Based on the literatures above, the research is found to be fragmaented for the estimation and synthesis of the basic accuracy of the robot. There are two reasons: firstly, no tolerance model that matchs with the error source can be applied to the robot structure model. Second, it is difficult for the complex mechanism, especially as robot, to achieve the tolerance optimization due to assembly process, batch differences, tolerance distribution forms. In the next section, after a statement of the SDT theory, Synthesis Method for the Structure Accuracy of Series Robot are investigated: first, an 3D tolerance model and the assembly constraint of the joint group are proposed; then The error operator is applied to the kinematic model of the robot, and sensitivity of the key error source is analyzed; lastly, the basic accuracy of the robot are improved by correct the key tolerances.

2 Robot Structure Accuracy Synthetic

2.1 SDT theory

Small displacement torsor (SDT) is a theoretical method for describing the tolerance zone with ideal shape features [11]. A small displacement torsor consists of three translational components and three rotation components, the small displacement spin SDT can be written as:

$$\tau = \begin{bmatrix} \psi & \Omega \end{bmatrix}^T \tag{1}$$

2.2 Component structure model

Dimensional tolerance model can be expressed with Jacobian spinor as:

$$\boldsymbol{\tau}_0 = \begin{bmatrix} [\boldsymbol{J}_{e1}] & \cdots & [\boldsymbol{J}_{en}] \end{bmatrix} [\boldsymbol{\tau}_{e1} \quad \cdots \quad \boldsymbol{\tau}_{en}]^T \tag{2}$$

where: $\boldsymbol{\tau}_0$ is a closed loop representing the assembly accuracy; $[\boldsymbol{\tau}_{e1} \quad \cdots \quad \boldsymbol{\tau}_{en}]^T$ represents n components of composition loops in the virtual chain, including internal function features and external function features; Jacobian matrices $[\boldsymbol{J}_{e1}] \quad \cdots \quad [\boldsymbol{J}_{en}]$ is used to describe the tolerance transmission between the composition loops; according to the Jacobian matrix definition, The jacobian matrix of the loops can be expressed as:

$$[\boldsymbol{J}]_{ei} = \begin{bmatrix} \left[\boldsymbol{R}_0^i\right]_{3\times3} \left[\boldsymbol{R}_{PTi}\right]_{3\times3} & \left[\boldsymbol{W}_i^n\right]_{3\times3} \left[\boldsymbol{R}_0^i\right]_{3\times3} \left[\boldsymbol{R}_{PTi}\right]_{3\times3} \\ \boldsymbol{0}_{3\times3} & \left[\boldsymbol{R}_0^i\right]_{3\times3} \left[\boldsymbol{R}_{PTi}\right]_{3\times3} \end{bmatrix}_{6\times6} \tag{3}$$

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where: $\left[\boldsymbol{R}_0^i\right]_{3\times3}$ denotes the direction component of the i-th characteristic node coordinate system relative to the base coordinate system; $\left[\boldsymbol{W}_i{}^n\right]_{3\times3}$ is the antisymmetric matrix representing the position component of the nth characteristic node coordinate system relative to the coordinate system i; $\left[\boldsymbol{R}_{PTi}\right]_{3\times3}$ is the direction matrix reprensenting the projection direction in the tolerance transfer process

2.3 Optimization method

If the robot kinematics is modeled by spinor, a unit error spinor can be expressed by $\$_{\mathbf{e}}=[\omega_e \quad v_e]^T$, The structural error spinor can be denoted as:

$$\$_{ei}=\begin{pmatrix}\omega_{ei} & r_{ei}\times\omega_{ei}+\dfrac{d_i}{\theta_{ei}}\omega_{ei}\end{pmatrix} \tag{4}$$

Where: θ_{ei} is the angle around the normal line between the ideal axis and the actual axis; d_{ei} denotes the vertical distance between the adjacent two axes;

The fact can be found that the tolerance zone built by a small displacement torsor is similar to the error spinor. If the tolerance zone $\boldsymbol{\tau}_0$ is defined as the limit value of the geometric error, the small movement spiral ξ_{ei} of the ith link coordinate system relative to the previous link is proposed by the transformation relationship $g_{e_{i-1}e_i}$ as:

$$g_{e_{i-1}e_i}=e^{\hat{\xi}_{i-1}\theta_{i-1}}g_{i-1,i} \tag{5}$$

Where: $g_{i-1,i}$ is the transformation from the coordinate system i-1 to the coordinate system i.

In the kinematics model of the robot established by the POE method, the mapping between the terminal pose and the tolerance field can be established:

$$\begin{aligned} g_{actual} &= \exp(\Delta\theta_{e1}\hat{\xi}_{e1}(0))\exp(\theta_1\hat{\xi}_1(0))\exp(\Delta\theta_{e2}\hat{\xi}_{e2}(0))\exp(\theta_2\hat{\xi}_2(0)) \\ &\quad \cdots\exp(\Delta\theta_{en}\hat{\xi}_{en}{}^\wedge(0))\exp(\theta_n\hat{\xi}_n(0))\exp(\hat{\xi}_B(0)) \end{aligned} \tag{6}$$

Where: θ_n is the parameter variable for the ideal joint n; $\Delta\theta_{en}$ is the parameter error for the actual joint n relative to the ideal joint, which is constant when only consider the static factors; $\xi_1{}^\wedge(0),\xi_2{}^\wedge(0),\cdots,\xi_n{}^\wedge(0)$ is the unit spinor for the robot ideal joint. $\xi_B(0)$ is the ideal pose of the tool coordinate system in the reference position.

For the robot is usually composed of multiple joints and links, the structure is complex. To achieve the accuracy synthesis of the robot structure, the model in eq.(6) need to be simplified. The method is decided to devide the whole robot into n joint group for analysis, the specific steps are as follows:

(1) Build the robot kinematic coordinate system, and divide the joint group. Each joint group contains one or more parts after assembly;

(2) Establish the tolerance loop for each joint group, and express each functional feature as a torsor form;

(3) Each functional feature is transimited to the joint axis to form a new small displacement torsor;

(4) The small displacement torsor is same as a small displacement of each joint, that is, the joint error;

(5) Establish a kinematic model with a robot error term;

(6) Optimize and comprehensively analyze to the tolerance zone of each functional feature.

3 Example validation

A PRR robot with 3 DOF consists of a upper arm, a lower arm, a wrist and an end effector. The coordinate system is established and the joint group is divided as shown in Figure 1.

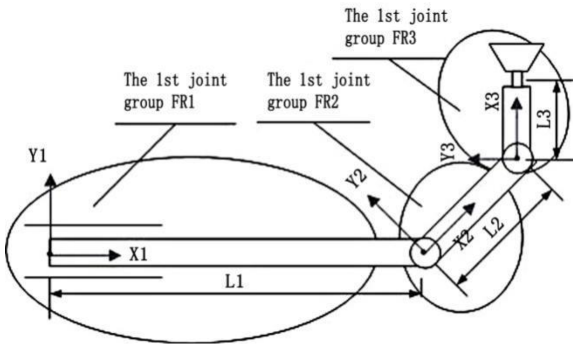

Fig. 1. The Division of groups of robot joints

The three-dimensional tolerance transfer model of the robot can be expressed as:

$$T_{actual} = \exp(FR_1)\exp(\theta_1\xi_1{}^{\wedge}(0))\exp(FR_2)\exp(\theta_2\xi_2{}^{\wedge}(0))\exp(FR_3)\exp(\theta_3\xi_3{}^{\wedge}(0))\exp(\xi_B{}^{\wedge}(0)) \quad (7)$$

Where $\boldsymbol{\theta_i}(\boldsymbol{i} = \mathbf{1}, \mathbf{2}, \mathbf{3})$ is the variable parameter of the three joints of the robot, $\boldsymbol{FR_i}$ is the total functional requirement of each joint group, which refers to the precision requirement. $\xi_1{}^{\wedge}(0), \xi_2{}^{\wedge}(0), \xi_3{}^{\wedge}(0)$ is the unit spinors for the ideal joints of the robot. $\xi_B(0)$ is the ideal pose of the tool coordinate system under the reference position.

Ignoring other factors on the terminal pose, only considering the tolerance of the key parts, the pose error of the end-effector of the robot can be expressed as:

$$\begin{aligned}\Delta T_e &= T_{actual} - T_{ideal} \\ &= \exp(\hat{\xi}_{e1})\exp(\theta_1'\hat{\xi}_1'(0))\exp(\hat{\xi}_{e2})\exp(\theta_2'\hat{\xi}_2'(0))\exp(\hat{\xi}_{e3})\exp(\theta_3'\hat{\xi}_3'(0))\exp(\hat{\xi}_B(0)) \\ &\quad -\exp(\theta_1\hat{\xi}_1(0))\exp(\theta_2\hat{\xi}_2(0))\exp(\theta_3\hat{\xi}_3(0))\exp(\hat{\xi}_B(0))\end{aligned} \quad (8)$$

According to the particle swarm interval optimization method, the optimal parameters are found in the algorithm. The learning factor is set to 2, the inertia weight w = 0.6, the number of initial population is set to 50, and the population size is 100.

The simulation results show that the optimal value of each joint group is: $t_z = 0.4347, t_2 = 0.3797, t_3 = 0.1412$

The next section is only described the optimization process of the second joint group.

The tolerance torsor of the second joint can be expressed as the small rotation error of $\boldsymbol{FR_2}$, $\boldsymbol{FR_2}$ is determined as the axis error of joint 2, and the tolerance zone is the

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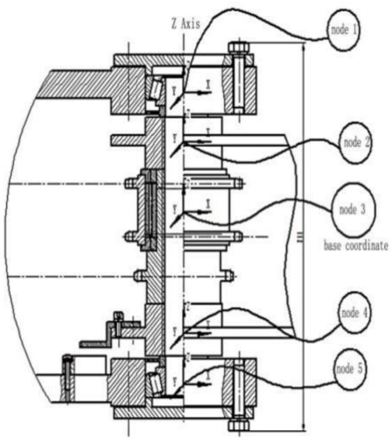
cylindrical surface whose axis is same as the joint axis. The tolerance zone $\boldsymbol{FR_2}$ can be expressed as :

$$\xi_{e2} = \begin{bmatrix} -\omega_{e2} \times v_{e2} \\ \omega_{e2} \end{bmatrix} = [\mathrm{J}_{e2}][v_2 \quad u_2 \quad 0 \quad \alpha_2 \quad \beta_2 \quad 1]^T \tag{10}$$

Where: $-t_2 / d_2 \le \alpha_2 \le t_2 / d_2$, $-t_2 / d_2 \le \alpha_2 \le t_2 / d_2$; $-t_2 / 2 \le u_2 \le t_2 / 2, -t_2 / 2 \le v_2 \le t_2 / 2$; $u_2^2 + v_2^2 \le t_2^2 / 4$, d_2 is the length of the application for the known tolerance zone, t_2 is the tolerance value for the tolerance zone to be determined.

In order to obtain the Jacobian spin model, the coordinate system is established firstly. The coordinate axes and the projection direction of the center of the tolerance zone of each part is shown in Figure 2. To avoid excessive projection conversion, all the coordinate system are set in the same direction as far as possible. Z axis is defined as the joint axis between the upper arm and the lower arm. The central axis of the arms is choiced as X axis. The direction of Y-axis follow the right-hand rule. Where $\mathbf{r_1}, \mathbf{r_2}, \mathbf{r_3}, \mathbf{r_4}, \mathbf{r_5}, \mathbf{r_6}$ are the processing variations of the parts itself, and $\boldsymbol{\theta_1}, \boldsymbol{\theta_2}, \boldsymbol{\theta_3}, \boldsymbol{\theta_4}, \boldsymbol{\theta_5}$ and $\boldsymbol{\theta_6}$ are assembly variation. $\boldsymbol{FR_2}$ is determined as the amount of variation of the actual axis relative to the theoretical axis at the second joint.



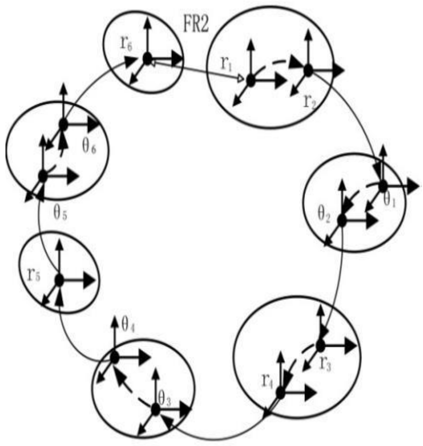


Fig. 2. The coordinate system of tolerance zone **Fig. 3.** The Functional relationships among the elements of the upper arm

The projection of each functional feature coordinate system to the global coordinate system is shown in Table 1.

Table 1. The orientation, position and projection matrices of functional features

node	functional feature	tolerance	direction matrix	position matrix	projection matrix
1	r_1 θ_1 θ_2	perpendicularity Ø52H8/k7 Ø52H8/k7	$R^1_{FR}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	$W^1_O=\begin{pmatrix}0&-74&0\\74&0&0\\0&0&0\end{pmatrix}$	$R_{PT1}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$
	$r_{2'}$	parallelism	$R^{1'}_{FR}=\begin{pmatrix}0&-1&0\\1&0&0\\0&0&1\end{pmatrix}$	$W^1_O=\begin{pmatrix}0&-74&0\\74&0&0\\0&0&0\end{pmatrix}$	$R_{PT1}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$
2	r_3	concentricity	$R^2_{FR}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	$W^2_O=\begin{pmatrix}0&-61&0\\61&0&0\\0&0&0\end{pmatrix}$	$R_{PT2}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$
	r_4	symmetry	$R^{2'}_{FR}=\begin{pmatrix}0&-1&0\\1&0&0\\0&0&1\end{pmatrix}$	$W^2_O=\begin{pmatrix}0&-61&0\\61&0&0\\0&0&0\end{pmatrix}$	$R_{PT2}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$
3	θ_3 θ_4	Ø52H8/k7 Ø38K7/h6	$R^1_{FR}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	$W^3_O=\begin{pmatrix}0&-18&0\\-18&0&0\\0&0&0\end{pmatrix}$	$R_{PT3}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$
4	r_5	perpendicularity	$R^1_{FR}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	$W^4_O=\begin{pmatrix}0&48&0\\-48&0&0\\0&0&0\end{pmatrix}$	$R_{PT4}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$
5	θ_5 θ_6 r_6	Ø52H8/k7 Ø26K7/h6 perpendicularity	$R^1_{FR}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	$W^5_O=\begin{pmatrix}0&74&0\\-74&0&0\\0&0&0\end{pmatrix}$	$R_{PT5}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$

From the eq.(8), the functional requirements is calculated according to the actual design tolerance: $t_{2a}=0.469mm$

According to Eq. 2, each functional feature involved in the modeling has effected on the whole functional requirements. The contribution of each functional feature can be acheived by the Monte Carlo method. It is assumed that all the functional features involved in the calculation are independent of each other, and the distribution type of the functional features is known. The small displacement of the functional requirement can be expressed as the sum of the small displacements of all the functional features involved in the base coordinate system:

$$\boldsymbol{FR}_0=\boldsymbol{FE}_1+\boldsymbol{FE}_2+\cdots+\boldsymbol{FE}_n \tag{11}$$

where: $\boldsymbol{FE}_i$ is the *ith (i = 1, 2, ..., n)* functional feature components for calculating the total functional requirements.

The contribution of each functional feature can be expressed as:

$$P_{i\delta}=\frac{\left|L^{+}_{i\delta}-L^{-}_{i\delta}\right|}{\sum_{i=1}^{n}\left|L^{+}_{i\delta}-L^{-}_{i\delta}\right|} \tag{12}$$

where $P_{i\delta}$ represents the contribution of the ith functional feature to the total functional requirement in the direction and position components, $L^{+}_{i\delta}$ and $L^{-}_{i\delta}$ is the upper and lower limit values of the i (i = 1, 2, ..., n) functional feature.

From the eq.(10), the Monte Carlo method is used to solve the contribution degree of the given tolerance term to the function requirement of the assembly. The result is shown in Table 2, compared with the optimization value obtained by the eq.(8).

From the Table.(2), the functional requirements calculated by the constituent loops of the design tolerances can not meet the accuracy of the kinematics of the robot. According to the contribution in the above table, the tolerance involved in nodes 1 and 5 are more sensitive to other nodes. The value of the perpendicularity is adjusted from 0.06 to 0.05 to improve a grate, and the fit H8 / k7 is changed by H7k6; on the other hand, the concentricity can be optimized from the non-standard tolerance to the standard tolerance of 0.05 for lower contribution. And then the result of recalcution is shown as Table 3.

Table 2. Contribution of each component ring in the 2nd joint group

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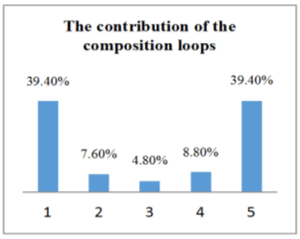
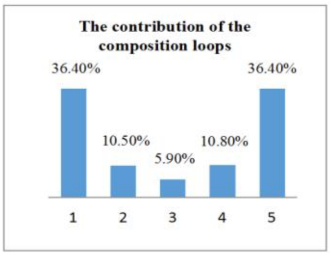
Node	Functional feature	tolerance	value(mm)	Contribution (%)	Contribution distribution
1	r_1	perpendicularity	0. 06	39. 4%	
	r_2	parallelism	0. 06		
	θ_1	Ø52H8/k7	0. 076		
	θ_2	Ø52H8/k7	0. 044		
2	r_3	concentricity	0. 04	7. 6%	
	r_4	symmetry	0. 12		
3	θ_3	Ø52H8/k7	0. 049	4. 8%	
	θ_4	Ø38K7/h6	0. 041		
4	r_5	perpendicularity	0. 06	8. 8%	
5	θ_5	Ø52H8/k7	0. 076	39. 4	
	θ_6	Ø26K7/h6	0. 044		
	r_6	perpendicularity	0. 06		
	FR	Joint gap	0. 469	100%	

The optimized FR are less than and close to the functional requirements of the kinematic robot. and the design tolerances are standardized according to the tolerance criteria. The optimization of the key tolerance project does not solve the accuracy problem completely, but can optimize the structural tolerance within a certain precision range. The basic accuracy of the robot is improved without increasing the manufacturing cost. Post-calibration step can be reduced from the design. And calibration time can be saved.

Table 3. Contribution of each component ring in the 2nd joint group

Node	Functional feature	tolerance	value(mm)	Contribution (%)	Contribution distribution
1	r_1	perpendicularity	0. 05	36. 40%	
	r_2	parallelism	0. 06		
	θ_1	Ø52H8/k7	0. 049		
	θ_2	Ø52H8/k7	0. 022		
2	r_3	concentricity	0. 05	10. 50%	
	r_4	symmetry	0. 12		
3	θ_3	Ø52H8/k7	0. 049	5. 9%	
	θ_4	Ø38K7/h6	0. 041		
4	r_5	perpendicularity	0. 06	10. 80%	
5	θ_5	Ø52H8/k7	0. 049	36. 40%	
	θ_6	Ø26K7/h6	0. 022		
	r_6	perpendicularity	0. 05		
	FR	Joint gap	0. 363	100%	

4 Conclusions

This method can be used to describe the mathematical model that express the relationship between the key tolerance term of the robot and the pose accuracy of end-effector of the robot.

The model can be used to prediction of the robot's body optimize and design. It can be used to optimize the key tolerance items to improve the basic accuracy of the robot.

The proposed method is helpful to realize the parametric design of the robot body structure accuracy. Based on the three-dimensional tolerance analysis to the joint group, the designer can be guided not only to modify the unreasonable tolerance items, but to take into account the processing cost.

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