Establishment of Model of Damping Mechanism for the Hard-coating Cantilever Plate

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Abstract: Constrained by the ability of testing damping, there is no effective method to separate the damping contribution of hard coating from the system damping of composite structure, which makes the establishment of model of damping mechanism become very difficult. In this paper, based on separating the damping contribution of hard coating, the method of creating the damping mechanism model of cantilever composite plate was studied. First of all, the cantilever plates before and after coating were tested and the dynamics characteristics parameters, such as natural frequency, damping ratio, vibration response and so on, were obtained. Moreover, from the analysis of the storage and dissipation energy in the uncoated and coated plate, the damping contribution of hard coating in the whole system damping was confirmed. Finally, based on the Lagrange equation, the dynamics model of cantilever composite plate was created considering both the material damping of hard coating and the remaining equivalent viscous damping of system. The correctness of model was verified by experiment results. The proposed method can be further applied to more complex structure and provide the reference to the study of vibration reduction mechanism and energy dissipation mechanism of hard coating.

Keywords: Hard coating; Cantilever thin plate; Damping mechanism; Analysis model; Basement excitation

1 Introduction

Hard coating can be used to reduce the vibration response of thin-walled structures in high temperature, high corrosion environment, and so broad attention has been arisen in recent years [1-3]. In order to better implement hard coating damping vibration, the damping mechanism of hard coating need to be acquired. Here, the damping mechanism is that using the model to explain the reason of vibration reduction of hard coating. On the basis of obtaining the damping mechanism, the optimization design of vibration reduction used hard coating can be executed. Now, most of the existing studies [4-8] on damping mechanism of hard coating come from micro material science. For example, Tassini [4] created a phenomenological model used to characterize the elastic properties of hard coating material, and the model can reproduce the basic features of the observed

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damping behavior of composite structure. To understand the specific mechanism of hard coating, Torvik [5] developed a slip damping model which can provide a satisfactory analytical representation of the damping and stiffness of hard-coating materials. Al-Rub[6] proposed a micromechanical theoretical and computational model to assess the main micromechanical mechanisms responsible for the experimentally observed phenomenon. From the above researches, most scholars think the internal friction among micro particles of hard coating is the cause of vibration reduction of hard-coating composite structure.

However, it is not enough to research the damping mechanism of hard coating completely based on micro materials science. It is necessary to create the dynamic models of the hard coating composite structures, and then research the damping mechanism of hard coating from the angle of vibration science. Some scholars have attempted to determine the vibration reduction mechanism of hard coating by creating dynamics model of composite structure. For example, Patsias [9] analyzed the dynamic properties of cantilever beam system partially coated with hard coating using the beam model, and predicted the nature characteristics and the damping parameters of the composite beam. Yen [10] developed an analytical procedure to study the dynamic behavior of beams coated with hard coating. But the status of current research is still unable to meet the needs of damping design of hard coating.

To create the analysis model which can accurately represent dynamic characteristics of hard coating composite structure, a lot of experimental results are needed, such as nature frequency, damping and vibration response for the uncoated and coated structures. However, compared with testing nature frequency and vibration response, there is no accurate method used to test damping [11]. The general method of identifying damping are the half power bandwidth [12] or free vibration attenuation method [13], which are all based on the assumption of equivalent viscous damping. The damping obtained by the two measure methods is an integrated value, usually named as damping ratio, which includes the material damping of hard coating, boundary conditions damping and fluid damping in air, etc. The damping contribution of hard coating is not effectively separated from the integrated value. Therefore, it is very difficult to create the damping mechanism model of hard-coating composite structure from the angle of vibration science.

In this research, the cantilever thin plate coated Mg-Al hard coating under the basement excitation was taken as study object, and the damping mechanism model of hard-coating composite structure was created based on obtaining the damping contribution of hard coating from the system damping of composite plate. In section 2, the cantilever plates before and after coating were tested and the dynamics characteristics parameters, such as natural frequency, damping ratio, vibration response and so on, were obtained. In section 3, from the analysis of the storage and dissipation energy in the uncoated and coated plate, the damping contribution of hard coating was separated. In section 4, the method of obtain the damping contribution of hard coating was proposed. Finally, in section5, based on the Lagrange equation, the dynamics model of cantilever composite plate was created considering both the material damping of hard coating and the remaining equivalent viscous damping of system, and the correctness of model was also verified by experiment.

2 The experiment of cantilever thin plate before and after coating

Mg-Al hard coating is a pseudo alloy coating, and it is usually used as anti-friction and anti-erosion coatings. This coating can also be used as damping coating because there are many cracks and voids in the coating (shown in Fig.1) and the internal friction among coating particles will produce damping effects. In this study, the titanium plate coated with Mg-Al hard coating in one side (shown in Fig.2) is chosen as experimental object.

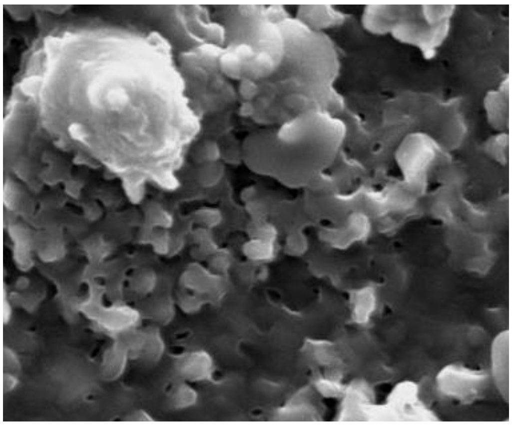

Fig.1. Surface morphology of Mg-Al hard coating

(a) uncoated plate (b)coated plate

Fig.2. The experiment object

The geometrical and material parameters of the plate are listed in Table.1, and the experiment system is shown in Fig.3. These parameters in Table.1 are obtained by the actual test or calculation. For example, vibration beam method [14] was adopted to identify the Young's modulus of Mg-Al coating. The plate is fixed on the fixture and clamping area is 20mm. The purpose of experiment is to obtain the natural frequency, damping ratio and vibration response of the cantilever plate before and after coating. The test equipments include LMS SCADAS acquisition front-end, King Design EM-1000F shaker and B&K 4517 light accelerometers, etc. It should be noted that it is true that the mass of accelerometer will impact the dynamics of system, but in this work, the mass impact is ignored. Because the mass of B&K 4517 accelerometer is only 0.6g, in addition, the accelerometer is placed on the lower part of cantilever plate, so the impact of accelerometer mass on the whole dynamics system is small and can be omitted.

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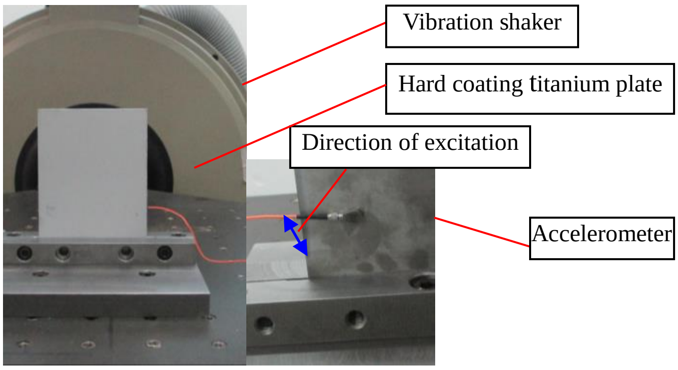

Fig.3. The coated thin plate under the base excitation

Table 1. Geometrical and material parameters of titanium plate and hard coating

Type of materials	Length (mm)	Width (mm)	Thickness (mm)	Young's modulus (GPa)	Density (kg/m3)	Poisson's ratio
Substrate Ti-6Al-4V	148	110	1.44	110.32	4420	0.3
Hard coating Mg-Al	148	110	0.02	49.2	3300	0.3

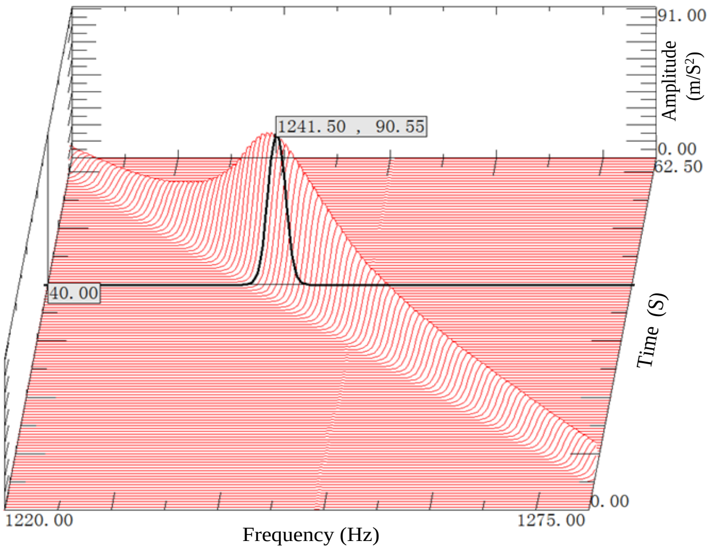

Fig.4. The 3D waterfall figure obtained by sweep excitation for the 6-order of coated plate

Table 2. The natural frequencies of uncoated and coated plate/Hz

Status \ Order	1	2	3	4	5	6
uncoated(A_1)	68.0	185.5	431.3	650.5	723.0	1228.0
coated(B_1)	66.5	182	412.5	616	715.3	1241.5
difference (B_1-A_1/A_1)	-2.2%	-1.8%	-4.3%	-5.3%	-1.1%	-1.1%

Table 3. The modal damping ratios of uncoated and coated plate/%

Status \ Order	1	2	3	4	5	6
uncoated(A_2)	0.37	0.19	0.22	0.29	0.15	0.11
coated(B_2)	0.57	0.24	0.24	0.43	0.25	0.19
difference (B_2-A_2/A_2)	54.1%	26.3%	9.1%	48.3%	66.7%	72.7%

Table 4. The vibration response of uncoated and coated plate/m/s^2

Status \ Order	1	2	3	4	5	6
uncoated(A_3)	169.7	13.2	636.8	63.1	29.1	835.1
coated(B_3)	121.9	12.2	593.3	41.2	14.2	569.4
difference (B_3-A_3/A_3)	-28.2%	-7.5%	-6.8%	-34.7%	-51.2%	-31.8%

The natural frequencies of cantilever plate are confirmed from 3D waterfall figure obtained by sweep excitation, an example about 3D waterfall figure can be seen in Fig.4, and the first 6 order natural frequencies of uncoated and coated plate are listed in Table 2. The method of half power bandwidth is adopted to obtain the modal damping ratios and the relevant testing results are listed in Table 3. The plate is excited by each order resonance frequency, excitation amplitude is 1g, when system comes to be steady-state, the resonant response of cantilever plate could be obtained and the results are listed in Table 4.

As can be seen from the test results, compared with uncoated plate, the natural frequencies of the plate coated with Mg-Al hard coating changes slightly，the biggest difference is less than 6%. For the first 6 orders, each order damping ratio of the coated plate increases and the resonant responses are reduced. Among them, the 1, 4, 5, 6 orders, changed significantly. It indicates that the hard coating possesses better effects of vibration reduction.

3 The analysis of storage and dissipation energy of thin plate before and after coating

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For the cantilever plate under base excitation shown in Fig.3, stored energies are primarily kinetic energy and potential energy, and dissipation energies include the frictional dissipation of clamping area of the plate, energy dissipation of material damping, and energy dissipation from air damping with movement of the plate. Under the same excitation amplitude, there are significantly different of the storage and dissipation energy between the uncoated and coated plate.

Set U_{b} as storage energy of each cycle of cantilever plate (substrate system), it can be expressed as:

$$U_{\mathrm{b}} = \hat{U}_{\mathrm{b}} = \hat{T}_{\mathrm{b}} \tag{1}$$

Where $\hat{U}_{\mathrm{b}}$ is maximum strain energy, $\hat{T}_{\mathrm{b}}$ is maximum kinetic energy.

Also, set D_{b} as dissipation energy of each cycle of cantilever plate, it can be expressed as

$$D_{\mathrm{b}} = D_{\mathrm{a}} + D_{\mathrm{m}} + D_{\mathrm{n}} \tag{2}$$

Where D_{a} is dissipation energy of clamping area of the cantilever plate, D_{m} is dissipation energy of material damping of the plate, D_{n} is dissipation energy from air damping.

Thus, the loss factor η_{b} of uncoated system can be represented as:

$$\eta_{\mathrm{b}} = \frac{D_{\mathrm{b}}}{2\pi U_{\mathrm{b}}} = \frac{D_{\mathrm{a}} + D_{\mathrm{m}} + D_{\mathrm{n}}}{2\pi U_{\mathrm{b}}} \tag{3}$$

If the cantilever plate system is in the resonance state under a certain natural frequency excitation, the loss factor of system is the modal loss factor, and modal loss factor is twice of the modal damping ratio, it can be written as:

$$\eta_{\mathrm{b}} = 2\xi_{\mathrm{b}} \tag{4}$$

Where ξ_{b} is the modal damping ratio of the uncoated plate, which is corresponded with the damping test values listed in Table.3.

After coating, the storage and dissipation energy will be change. Set U_{s} as storage energy of each cycle of coated plate and it can be expressed as:

$$U_{\mathrm{s}} = U'_{\mathrm{b}} + U_{\mathrm{c}} \tag{5}$$

Where U'_{b} is storage energy of the substrate, U_{c} is storage energy of the hard coating.

Similar with the analysis of uncoated plate, D_{s} is set as dissipation energy of coated plate and can be expressed as:

$$D_{\mathrm{s}} = D'_{\mathrm{a}} + D'_{\mathrm{m}} + D'_{\mathrm{n}} + D_{\mathrm{c}} \tag{6}$$

Where $D'_{\mathrm{a}}, D'_{\mathrm{m}}, D'_{\mathrm{n}}$ are the dissipated energies of clamping area, the substrate material and air damping respectively and D_c is dissipation energy of hard coating material.

Similarly, the loss factor η_{s} of the coated system can be written as:

$$\eta_{\mathrm{s}} = \frac{D_{\mathrm{s}}}{2\pi U_{\mathrm{s}}} = \frac{D'_{\mathrm{a}} + D'_{\mathrm{m}} + D'_{\mathrm{n}} + D_{\mathrm{c}}}{2\pi (U'_{\mathrm{b}} + U_{\mathrm{c}})} \tag{7}$$

In the resonance state, the loss factor η_s is modal loss factor. The relationship between the modal loss factor η_s and modal damping ratio ξ_s is:

$$\eta_s = 2\xi_s \tag{8}$$

Where ξ_s is the modal damping ratio of coated plate, which is also corresponded with the damping test values listed in Table.3.

Eq.(7) can be further expressed as:

$$\eta_s = \frac{D_c / U_c}{2\pi(U'_b / U_c + 1)} + \frac{(D'_a + D'_m + D'_n) / U'_b}{2\pi(U_c / U'_b + 1)} \tag{9}$$

If set

$$\frac{(D'_a + D'_m + D'_n)}{2\pi U'_b} = \frac{D_a + D_m + D_n}{2\pi U_b} \tag{10}$$

It can yield:

$$\eta_s = \frac{\eta_c}{U'_b / U_c + 1} + \frac{\eta'_b}{U_c / U'_b + 1} = \frac{\eta_c}{U'_b / U_c + 1} + \frac{\eta_b}{U_c / U'_b + 1} \tag{11}$$

Where η_c describes the contribution of hard coating in the whole loss factor of system (or the whole system damping). Here, η'_b is named as the remaining equivalent viscous damping of system, because it excludes the effect of the material damping of hard coating. It can be seen that the damping contribution of hard coating has been separated from the whole system damping.

In the subsequent analysis model, it can be assumed that there only includes the material damping η_c of hard coating (Generally, the material damping of substrate is much less than hard coating.) and the remaining equivalent viscous damping η'_b. Referring to the Eq.(10), it can be thought $\eta'_b = \eta_b$, that is, the system damping η_b of uncoated plate can be used in the analysis model of coated plate. It is worth noting that the meaning of loss factor η_c comes from macro vibration science, but it is almost consistent with the definition of material science (shown in Eq.(19) of section 5), only, for the different order of thin plate, η_c will be assigned different values. As for η_b, it is different with the definition of material science, because it includes not only material damping of substrate but also other damping, such as frictional damping in the clamping area, air damping, etc, so it is named as the remaining equivalent viscous damping in this work.

4 Obtain the damping contribution of hard coating in the whole system damping

By defining the storage energy ratio $R = U_c / U'_b$ between hard coating and substrate, Eq.(11) can be expressed as:

$$\eta_c = \frac{\eta_s(R+1) - \eta_b}{R} \tag{12}$$

Thus, as long as the storage energy ratio R is known, the damping contribution of hard coating in the whole system damping can be obtained.

According to Eq.(1), the storage energy ratio R can also be expressed as the maximum strain energy ratio or maximum kinetic energy ratio, and is shown as follow:

$$R = \frac{\hat{U}_{\mathrm{c}}}{\hat{U}'_{\mathrm{b}}} = \frac{\hat{T}_{\mathrm{c}}}{\hat{T}'_{\mathrm{b}}} \tag{13}$$

Where $\hat{U}_{\mathrm{c}}$ and $\hat{U}'_{\mathrm{b}}$ are the maximum strain energy of the hard coating and substrate respectively, and $\hat{T}_{\mathrm{c}}$ and $\hat{T}'_{\mathrm{b}}$ are the maximum kinetic energy of the hard coating and substrate respectively.

Because energy can be superimposed, the total strain energy and kinetic energy of the hard coating composite structure are:

$$\hat{U}_{\mathrm{s}} = \hat{U}_{\mathrm{c}} + \hat{U}'_{\mathrm{b}} \tag{14}$$

$$\hat{T}_{\mathrm{s}} = \hat{T}_{\mathrm{c}} + \hat{T}'_{\mathrm{b}} \tag{15}$$

According to the Rayleigh quotient, the square of natural frequency of structure system is equal to the ratio of the maximum potential energy (Here, the strain energy is equivalent to potential energy) and the maximum kinetic energy, thus it can yield:

$$\frac{f_{\mathrm{s}}^2}{f_{\mathrm{b}}^2} = \frac{\hat{U}_{\mathrm{s}} / \hat{T}_{\mathrm{s}}}{\hat{U}_{\mathrm{b}} / \hat{T}_{\mathrm{b}}} = \frac{1 + \hat{U}_{\mathrm{c}} / \hat{U}'_{\mathrm{b}}}{1 + \hat{T}_{\mathrm{c}} / \hat{T}'_{\mathrm{b}}} \tag{16}$$

Where f_{b} and f_{s} are the natural frequencies of uncoated and coated plate.

In Eq. (16), the ratio of kinetic energy can also be expressed as the ratio of mass, that is:

$$\hat{T}_{\mathrm{c}} / \hat{T}'_{\mathrm{b}} = H_{\mathrm{c}} \rho_{\mathrm{c}} / H_{\mathrm{b}} \rho_{\mathrm{b}} \tag{17}$$

Where H_{c} and H_{b} are the thickness of hard coating and substrate respectively, and ρ_{c} and ρ_{b} are the density of hard coating and substrate. So, Eq. (16) can be expressed as:

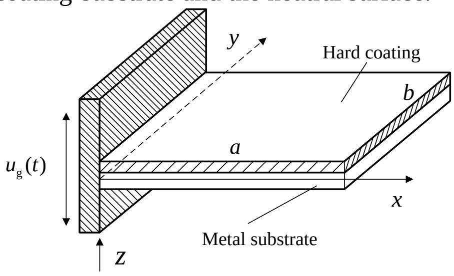
$$R = \frac{\hat{U}_{\mathrm{c}}}{\hat{U}'_{\mathrm{b}}} = \frac{f_{\mathrm{s}}^2}{f_{\mathrm{b}}^2} \left(1 + \frac{H_{\mathrm{c}} \rho_{\mathrm{c}}}{H_{\mathrm{b}} \rho_{\mathrm{b}}}\right) - 1 \tag{18}$$

Eq.(18) shows that if getting the natural frequency of uncoated and coated plate , the thickness and density of hard coating and substrate, the storage energy ratio can be obtained. Furthermore, the damping contribution of hard coating corresponding each order loss factor of system can be obtained by substituting the Eq.(18) into Eq.(12).

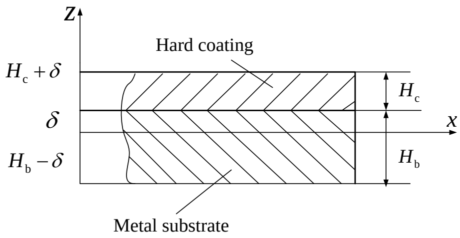
5 Creating the damping mechanism model of cantilever composite plate

5.1 Analysis of hard-coating composite plate under base excitation

A cantilever thin plate with hard coating on one side is shown in Fig.5(a). Its length is a and the width is b. The thin plate is subjected to the base excitation denoted by $u_{\mathrm{g}}(t)$ at its clamping end. Fig. 5(b) shows its cross section and set xy coordinate plane locate in the neutral surface. Where δ is the distance between the interface bonding of coating-substrate and the neutral surface.



(a) Hard-coated composite plate



(b) Cross section of composite plate

Fig. 5. Hard-coated composite plate under base excitation

The elastic modulus of hard coating is expressed as complex modulus and shown as:

$$E_c^* = E_c(1 + i\eta_c) \tag{19}$$

Where * refers to complex value, E_c^* are the complex modulus of the hard coating, and E_c, η_c are the corresponding Young's modulus (or storage modulus) and the loss factor respectively. For the normal material, the η_c is usually constant, but here η_c is substituted according to every order, that is, inputting the identified value shown in Eq.(12). This reflects the material damping of hard coating on the contribution of each order modal of composite plate.

Then, the complex shear modulus of hard coating can be expressed as:

$$G_c^* = \frac{E_c^*}{2(1+\mu_c)} \tag{20}$$

Where G_c^* is the complex shear modulus of hard coating, and μ_c is corresponding Poisson's ratio.

The distance δ between the interface and neutral surface can be determined as [15]:

$$\delta = \frac{E_b H_b^2 - E_c H_c^2}{2\left(E_b H_b + E_c H_c\right)} \tag{21}$$

Where E_b is Young's modulus of metal substrate.

It is assumed that the base excitation at the clamping end of the plate is harmonic and expressed as:

$$u_g(t) = U_G e^{i\omega t} \tag{22}$$

Where U_G is the amplitude of base excitation and ω is the angular frequency. Hence, the total displacement of an arbitrary point on the cantilever plate can be determined as:

$$\lambda(x,y,t) = u_g(t) + w(x,y,t) \tag{23}$$

Where the $w(x,y,t)$ refers to the deflection of any point.

If the hard-coated composite plate satisfies the basic assumption of the thin plate theory, thus the strain energy of the plate is

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$$\begin{aligned}\hat{U}_s &= \frac{1}{2}\iiint_V \left(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy}\right)\mathrm{d}V \\ &= \frac{1}{2}\iint_A \left\{ L_1\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2\right] + 2L_2\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + L_3\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \right\}\mathrm{d}A\end{aligned} \tag{24}$$

Where *V*,*A* is the volume and area of the plate and L_1, L_2 , L_3 are

$$L_1 = \frac{E_\mathrm{b} N_1}{1-\mu_\mathrm{b}^2} + \frac{E_\mathrm{c}^* N_2}{1-\mu_\mathrm{c}^2} \qquad L_2 = \frac{E_\mathrm{b} N_1 \mu_\mathrm{b}}{1-\mu_\mathrm{b}^2} + \frac{E_\mathrm{c}^* N_2 \mu_\mathrm{c}}{1-\mu_\mathrm{c}^2} \qquad L_3 = 4G_\mathrm{b} N_1 + 4G_\mathrm{c}^* N_2$$

$$N_1 = \frac{H_\mathrm{b}^3}{12} + H_\mathrm{b}\left(\frac{H_\mathrm{b}}{2} - \delta\right)^2 \qquad N_2 = \frac{1}{3}H_\mathrm{c}^3 + \delta\left[H_\mathrm{c}^2 + \delta H_\mathrm{c}\right] \tag{25}$$

Where G_b , μ_b are shear modulus and Poisson's ratio of metal substrate respectively.

Eq.(24) and Eq.(25) show that the energy dissipation due to the material damping of hard coating has been included in the calculation of strain energy of system.

The kinetic energy of composite plate is:

$$\hat{T}_s = \frac{1}{2}\left(\rho_\mathrm{b} H_\mathrm{b} + \rho_\mathrm{c} H_\mathrm{c}\right)\iint_A \left(\frac{\partial w}{\partial t}\right)^2 \mathrm{d}A \tag{26}$$

According to the two-dimensional beam function method, the deflection of the hard-coated composite plate can be approximately expressed as:

$$w(x,y,t) = \sum_{r=1}^{R}\sum_{s=1}^{S} X_r(x) Y_s(y) a_{rs}(t) \tag{27}$$

Where $X_r(x)$ refers to the *r*-order modal shape function of clamped-free beam, $Y_s(y)$ refers to the *s*-order modal shape function of free -free beam, $a_{rs}(t)$ is the contribution coefficient of each beam function, and *R*, *S* are the upper limits of mode interested.

Set $L = \hat{T}_\mathrm{s} - \hat{U}_\mathrm{s}$,applying Lagrange equations yields:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{a}_{pk}(t)} - \frac{\partial L}{\partial a_{pk}(t)} + \frac{\partial D}{\partial \dot{a}_{pk}(t)} = 0 \qquad \begin{pmatrix} p = 1,2,3,\cdots,R \\ k = 1,2,3,\cdots,S \end{pmatrix} \tag{28}$$

Where *D* is the dissipation function of the composite plate which has excluded the contribution of hard-coating material, it could also be seen as dissipation energy of the uncoated plate, that is:

$$D = D_\mathrm{b} = \frac{1}{2} c_j \sum_{r=1}^{R}\sum_{s=1}^{S} (X_r(x) Y_s(y) \dot{a}_{rs}(t))^2$$

Where c_j is the damping coefficient of the *j*-order, it can be expressed as:

$$c_j = \eta_{b,j}\omega_j / m \tag{29}$$

Where $\eta_{b,j}, \omega_j$ are the remaining equivalent viscous damping and natural frequency of the *j*-order of uncoated plate respectively, *m* is the mass of per unit area.

Because the plate is excited by a single frequency, it can be assumed that

$$a_{rs}(t) = a_{rs} e^{\mathrm{i}\omega t} \tag{30}$$

Then, the deflection of the hard-coating composite plate can be re-written as:

$$w(x, y, t) = \sum_{r=1}^{R} \sum_{s=1}^{S} X_r(x) Y_s(y) a_{rs} e^{\mathrm{i}\omega t} \tag{31}$$

According to Eq. (28), a set of linear algebraic equations can be obtained, in matrix form:

$$\left(\boldsymbol{K} + \mathrm{i}\boldsymbol{C}_1 - \omega^2 \boldsymbol{M} + \mathrm{i}\omega \boldsymbol{C}_2\right)\boldsymbol{a} = \boldsymbol{q} \tag{32}$$

Where

$$\boldsymbol{a} = \left[a_{11} \quad \cdots \quad a_{1S} \quad a_{21} \quad \cdots \quad a_{2S} \quad \cdots \quad a_{rs} \quad \cdots \quad a_{RS}\right]^T \tag{33}$$

represents the response of system, the coefficient matrix $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{C}_1$ is the material damping matrix of hard coating, $\boldsymbol{C}_2$ is the remaining equivalent viscous damping of system,$\boldsymbol{q}$ is the vector of the excitation force.

From the Eq.(32), the response vector a can be obtained corresponding to a specific exciting frequency. Substituting a to the Eq.(27), the deflection of any point at the hard-coated composite plate can be acquired. Finally, the vibration response under base excitation is achieved through Eq. (23).

5.2 Solution of vibration response

Corresponding to the experiment shown in section 2, the created analysis model is used to solve the vibration response. By comparing with the experimental result, the correctness of model can be verified.

The each order storage energy ratio between hard coating and substrate can be obtained by substituting the natural frequencies of uncoated and coated plate into the Eq.(18). Moreover, substituting storage energy ratios into Eq.(12), the damping contribution of hard coating in each order system damping can be obtained. All the results are listed in Table.5.

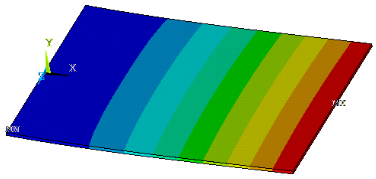
Table 5. The first 6 orders storage energy ratios and the damping contribution of hard coating in each order system damping

Order	1	2	3	4	5	6
storage energy ratio R	0.0193	0.0187	0.0189	0.0190	0.0189	0.0188
Modal loss factor η_c /%	0.2186	0.0581	0.0259	0.3655	0.4878	0.1308

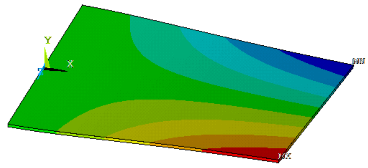
Then, substituting the first 6 orders modal damping ratios of uncoated plate listed in Table.3 and the first 6 orders loss factors of hard coating listed in Table.5 into the analysis model, the vibration responses of coated plate are obtained. Here, excitation amplitude is still 1g in the analysis model and the vibration picking point is consistent with experiment, the coordinate is : x=0.032m,y=0.028m. The obtained analysis results are listed in Table.6. It can be seen that there are only a small amount of differences between the experimental results and analysis results obtained by the created model and the maximum difference is less than 6.5%(corresponding to the 5th order) , so the correctness of the created model is verified. In addition, it should be explained that the second order and the fourth order resonant responses can not be solved by the created model, because the model can not effectively simulate the torsion excitation. The first 6 orders modal shapes of coated cantilever plate are shown in Fig.6, which are obtained by finite element method used ANSYS software. It can be seen that the 2nd and 4th modal shape are torsion vibration.

Table 6. Resonant response comparison between experiment and analysis for the coated cantilever plate/ m/s^2

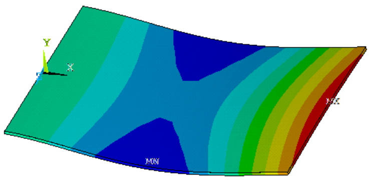
Status \ Order	1	2	3	4	5	6
experiment（A_4）	121.9	12.2	593.3	41.2	14.2	569.4
analysis（B_4）	114.8	-	576.5	-	15.1	568.5
$\lvert A_4 - B_4 \rvert / A_4$	5.82%		2.83%		6.33%	0.158%



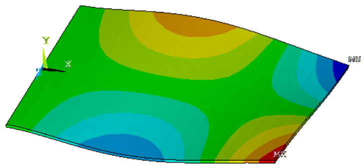
(a) the 1st order



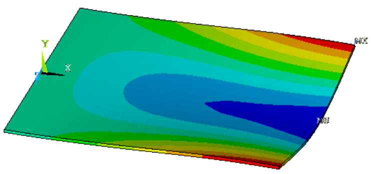
(b) the 2nd order



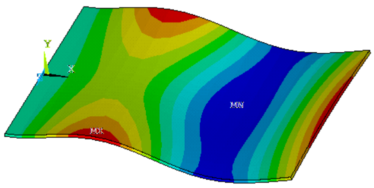
(c) the 3rd order



(d) the 4th order



(e) the 5th order



(f) the 6th order

Fig.6. The first 6 orders modal shapes of coated cantilever plate

6 Conclusions

Based on separating the damping contribution of hard coating from the system damping obtained by test, the method of creating the damping mechanism model of cantilever composite plate was studied and reached the following conclusions:

(1) The cantilever thin plate coated with Mg-Al hard coating on one side was tested, the test results show each order damping ratio increases and vibration responses are reduced and some orders changed significantly. It indicates that the hard coating possesses better effects of vibration reduction.

(2) By analyzing the storage and dissipation energy of composite structure and utilizing the definition of loss factor, the damping contribution of hard coating can be separated from the whole system damping and the relevant formula was derived.

(3) The damping of coated composite plate system includes the material damping of the hard coating and other damping that can be named as remaining equivalent viscous damping. In addition, the remaining equivalent viscous damping can be replaced by the system damping of uncoated plate. The practice indicates that it could not cause a large analysis error.

(4) Utilizing the material damping of hard coating and the remaining equivalent viscous damping, the analytical model of hard-coating composite plate can be created, which can effectively simulate the dynamics of the real structure. The calculation results of the first 6 orders resonant response (except for the 2 and 4 orders) demonstrate that the maximum error is less than 6.5%.

For the created analysis model in this study, the damping contribution of hard coating was separated from the system damping of composite plate, and then the goal of creating the model of damping mechanism from vibration science was achieved. Furthermore, by analyzing the impacts of thickness, Young’s modulus and loss factor of hard coating on the effect of vibration reduction, the created model also can provide the reference for choosing and fabricating the hard-coating materials, so parameters analysis should be done in next work.

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