Traveling wave analysis of rotating disk with viscoelastic material treatment

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Abstract: In present work, the dynamical modeling method of the disk with viscoelastic damping layer and the influence of viscoelastic damping material on its traveling wave vibration characteristics are studied. Firstly, the finite element model of the central fixed disk with the viscoelastic damping materials treatment was established. Secondly, modal analysis of the stationary disk with viscoelastic material was conducted. Compared with the results of experimental test, the accuracy of the modeling method was verified. Finally, based on the finite element model of the disk with viscoelastic damping layer established in this paper, the affect of rotational speed on the natural characteristics of damping structure was calculated, and the influence of viscoelastic damping layer on the traveling wave curves of different mode shape of thin disk were compared. The present results show that the viscoelastic damping layer can affect the natural frequency and traveling wave frequency of the disk significantly, but the order of vibration mode remains unchanged.

Key words: Rotating disk; Natural frequency; Viscoelastic damping material treatment; Traveling wave vibration

1 Introduction

Many kinds of disk structures are widely used in rotating machineries such as the gas turbines and aircraft engines and so on. Due to high rotating speeds, the disk structures always suffer the high centrifugal loads and many other kinds of impulse load. Since large vibration amplitude, especially resonance, occurs easily, it will influence the working performance of the device. Thus, the vibration analyses of rotating thin disk structures are often indispensable. In many cases, the optimization of structural design cannot avoid the vibration. Thus, attaining vibration suppression by damping treatment is very important. For example, the thin disk with viscoelastic damping layer can significantly improve the overall damping capacity, and it is an

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effective way of the vibration reduction. Therefore, it is of great value to investigate the viscoelastic damping modeling and dynamic characteristic analysis theoretically and practically for the thin disk structures.

Southwell [1] first analyzed the traveling wave characteristics of the rotating disk incomplete clamped at its center. Based on the classical thin plate theory, Chonan et al. [2] improved the vibration mode function of center fixed rotating disk by boundary conditions using the hypothesis vibration mode function method and got Campbell FIG and analyzed the traveling wave phenomenon of disk vibration.

Due to simple structural form, wide control frequency, high reliability and low cost, viscoelastic structures are widely used in ship, submarine and aerospace structures for vibration control and noise reduction. Viscoelastic damping materials, with both elastic and viscous properties, belong to the polymer materials. The strain always lags behind the change of the stress under the action of alternating stress, this process transforms external force into heat energy and dissipates, that is viscoelastic material tends to have high damping capacity. The storage modulus and loss factor of viscoelastic material are both functions of temperature and frequency. For most of polymer viscoelastic material, there is a certain equivalent relation between the effect of temperature and frequency on the mechanical properties. The constitutive equations of viscoelastic material mainly include the generalized Maxwell model, the generalized Kelven model and the GHM model proposed by Golla and Hughes, and the fractional derivative model proposed by Bagley etc [3-5]. The methods of analyzing the dynamic characteristics of viscoelastic material composite structure are the complex stiffness method, direct frequency response method and the modal strain energy method [6].

In the last decades, many investigations on viscoelastic damping treatment have been carried out. For example, Kerwin, Mead, Rao, Miles et al. [7-11] investigated the beam structure with constrained damping layer treatment. He et al. [12] proposed the method to calculate the natural frequency and loss factor of the CLD beam. Yim et al. [13] studied the dynamic response of viscoelastic beams, Yang et al. [14] investigated the vibration characteristic and dynamic stability of traveling sandwich beam using the finite element method. Cortés and Elejabarrieta [15] analyzed the vibration of beam, unconstrained damped beam and beam structure with constrained damping layer. In recent years, the research on the shells with viscoelastic damping layer has gradually increased. Johnson and Keinholz [16] proposed a modal strain energy method to solve the natural frequency and the loss factor of the shell with constrained damping layer. Based on the finite element method (FEM), Hu, Huang, Sainsbury, Masti [17-19] analyzed vibration behaviors and damping effect of a viscoelastic sandwich cylindrical shell. Then, Shariyat [20-25] expended the FE model to perform the dynamic buckling analysis using a double-superposition global–local theory for the sandwich/multilayer shells and plates. Kumar and Singh [26] investigated the vibration and damping characteristics of curved panel treated with constrained viscoelastic layer. Manconi [27] imported a spectral finite element method (SFEM) to analyze the elastic guided waves in composite viscoelastic plates. Taupin [28] investigated the dispersion and damping characteristics of viscoelastic laminate plates via a wave finite element method. The dynamic control equation for a rotating laminated circular plate was derived by Li et al. [29] adapting to Hamilton principle. Mace [30] extended the WFE approach to two-dimensional homogeneous structures.

Although the vibration characteristics of the rotating disk under center fixed constraints can be obtained using the classical theory of elasticity, unfortunately, to the authors' knowledge, there is less mature elastic mechanics analytical theory to analyze the thin disk structure with viscoelastic damping layer so far. In the research of vibration reduction and dynamic design for engineering application, the finite element method is a powerful method to handle the natural characteristics of the thin disk with viscoelastic damping layer and the traveling wave characteristics with the rotation influence considered.

In this paper, experimental and the finite element methods were introduced to analyze the traveling wave vibration characteristics of center fixed disk with viscoelastic damping material treatment. Firstly, the finite element method of viscoelastic damping material was introduced, and then the model of the center fixed thin disk with viscoelastic damping layer treatment was established based on the finite element method, meanwhile, modal test was imported for the analyze the static viscoelastic damping layer-disk structure and the accuracy of the finite element method was verified. Further, some numerical examples were presented to confirm the influence of rotating speeds and the viscoelastic damping layer on the frequency of disks

2 The modeling of the thin disk with viscoelastic damping layer treatment

2.1 The simulation of viscoelastic damping materials

The constitutive relations model of viscoelastic damping material is very important. When viscoelastic damping modeling using the finite element method, not only accurately expresses the dynamic mechanical properties of viscoelastic damping materials is required, but also the model should be easy to solve as well. Present constitutive relations models of viscoelastic damping materials are mostly based on theoretical, empirical, experimental and numerical simulation, which mainly including the following three types.

Prony series model

Using Prony series, the basic form of viscoelastic property are expressed as

$$G(t) = G_{\infty} + \sum_{i=1}^{n_G} G_i \exp\left(-\frac{t}{\tau_i^G}\right) \tag{1}$$

$$K(t) = K_\infty + \sum_{i=1}^{n_K} K_i \exp\left(-\frac{t}{\tau_i^K}\right) \quad (2)$$

where K_∞ , K_i are bulk modulus, G_∞ , G_i are shear modulus, τ_i^G , τ_i^K are relaxation.

In ANSYS, in the element entry for viscoelastic material modeling, the relative volume relaxation modulus α_i^K and relative shear relaxation modulus α_i^G are

$$\alpha_i^G = G_i / G_0 \quad (3)$$

$$\alpha_i^K = K_i / K_0 \quad (4)$$

where G_0 , K_0 are the instantaneous modulus of the viscoelastic damping material and they are defended as follows

$$G_0 = G(t=0) = G_\infty + \sum_{i=1}^{n_G} G_i \quad (5)$$

$$K_0 = K(t=0) = K_\infty + \sum_{k=1}^{n_k} K_i \quad (6)$$

For the viscoelastic damping materials, the Poisson's ratio is a function of time, that is $\mu = \mu(t)$. As for the condition where Poisson's ratio can be approximated as

$$G(t) = \frac{E(t)}{2(1+\mu)} \quad (7a)$$

$$K(t) = \frac{E(t)}{3(1-2\mu)} \quad (7b)$$

where $E(t)$ is relaxation modulus.

Complex modulus model

The plural form is used to express the dynamic elastic modulus of the viscoelastic material for complex modulus model and the constitutive relationship of complex modulus model is

$$\sigma = G^* \varepsilon = G^{'}(1+i\eta)\varepsilon \quad (8)$$

The complex modulus is defined as

$$G^* = G^{'} + \mathrm{i}G^{"} = G^{'}(1+\mathrm{i}\eta) \quad (9)$$

where $G^{'}$ is the real part of complex modulus, called storage modulus, $G^{"}$ is the imaginary part of complex modulus, called loss modulus, $\mathrm{i} = \sqrt{-1}$ is the unit of imaginary part, η is the loss factor of the viscoelastic damping material

In present analysis the loss factor of the viscoelastic damping material is defined by

$$\eta = G^{"} / G^{'} \quad (10)$$

In the complex modulus model, all the quantities are constant, and it has not considered the frequency dependent characteristic of viscoelastic damping material, which making the form simple. Thus, it is suitable for the situation of small frequency-dependent and can characterize the dynamic mechanical properties of viscoelastic materials well under harmonic excitation.

Index model

The definition of index model can be expressed as

$$E_c(\omega) = E_d + \frac{c}{t_0}\frac{i\omega t_0}{1+i\omega t_0} \tag{11}$$

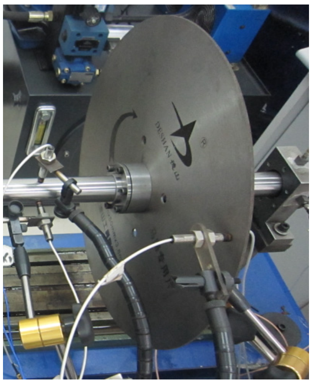
where E_d is relaxation modulus, c is viscosity, and its unit is $\mathrm{Pa\cdot s}$, which means the resistance in per unit area when shear rate is $1/\mathrm{s}$, t_0 is relaxation time.

Finite element model

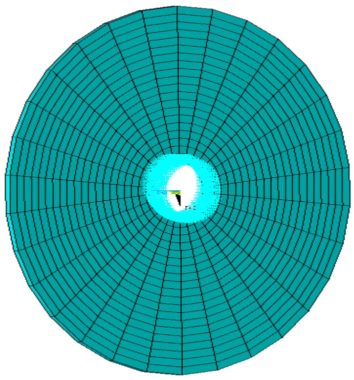
The considered thin disk is in Fig. 1(a) and the geometrical parameters and material parameters are all listed in Table 1.

Tab. 1. Material and geometrical parameters of thin disk with viscoelastic damping layer

sign	symbol description	basis disk	viscoelastic damping layer Zn33	units
a	outer radius	220	220	mm
b	inner radius	25	25	mm
h	thickness	2.8	1.2	mm
E	Young's modulus	206	2.68+0.968i	GPa
μ	Possion's ratio	0.3	0.49	
ρ	density	7800	930	kg/m3



(a) Photo of the disk specimen



(b) Finite element model of the disk

Fig. 1. The thin disk

2.2 Finite element model

In order to verify the accuracy and rationality of the proposed method in this paper, the natural characteristics of the stationary thin disk are calculated firstly. The calculation results obtained by the finite element model are verified by experiment. In the finite element calculations, the element of Solid95 was chosen to create the FE model and the whole model consists of 600 elements. The constrained conditions of thin disk are central fixed. The command stream is NSEL,S,LOC,X,r1 used in ANSYS, finite element model of the thin disk is shown in Fig. 1.(b).

The energy dissipation mechanism of composite material damping structure is mainly shear deformation of damping material. Thus in the finite element modeling, the calculation method must be able to reflect the structure of the shear strain energy truthfully. The object of this paper is a layered disk and SOLID95 element can satisfy the need of boundary curve. SOLID95 element is the higher order form of Solid45 (shown in Fig.2), which has 20 nodes; each node has three degrees of freedom. SOLID95 element allows the irregular shape without reducing the accuracy, especially suitable for the curved boundary models and the offset shape compatibility is good.

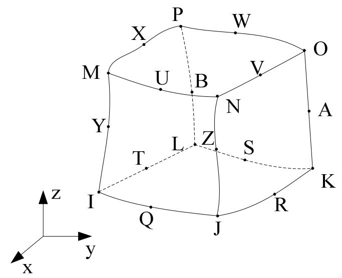


Fig. 2. The unit schematic model of SOLID95

3 Numerical results and discussion

3.1 The influence of different viscoelastic model on the natural frequency of thin disk

In order to verify the rationality of the finite element method and the obtained results, hammering method modal test is performed for the stationary disk specimen. The obtained natural frequencies are compared with the ANSYS calculation results for confirming the effectiveness of the established model. In the finite element method, viscoelastic material parameters were inputted to analysis model using two different input methods, direct input of material damping and the Prony series method, and the results are shown in Table 2.

From Table 2, it can be found obviously that the calculation errors of the natural frequency of the (0,1) order, the (0,2) order, the (0,3) order and above are 17.7% , 7.35% and less than 17% respectively, the natural frequencies of the (0,3), (0,4), (1,1),

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(0,5), (0,6) order are relatively closer and have high accuracy. The comparison shows that the developed method is suitable for analysis and calculation of viscoelastic free damping layer Zn33. The results obtained by two different input methods are close in the low-order, such as (0,1), (0,2), (0,3) order; as for high-order, such as the (0,5), (1,2), (0,6) order, the results of Prony series method are closer to the experimental results than those of direct input of material damping method.

Tab. 2. Tests and calculation results of natural frequency of the disk with viscoelastic damping layer

(*m*, *n*)	Natural frequency/Hz			damping ratio
	Experimental	Complex modulus	Prony series	ANSYS
(0,1)	50.00	58.86	59.03	5.2×10^{-6}
(0,2)	78.13	83.88	85.59	3.8×10^{-6}
(0,3)	175.00	175.28	178.99	4.6×10^{-6}
(0,4)	303.13	309.99	316.16	7.3×10^{-6}
(1,0)	356.25	403.79	404.18	7.8×10^{-6}
(1,1)	462.50	435.50	436.59	8.2×10^{-6}
(0,5)	496.88	483.39	492.64	1.2×10^{-5}
(1,2)	650.00	545.83	548.92	9.8×10^{-6}
(0,6)	721.88	698.11	711.14	2.2×10^{-5}

3.2 The influence of viscoelastic damping layer on the natural frequency of thin disk

The finite element model of thin disk with viscoelastic free damping layer is shown in Fig. 3. In the static state, the natural characteristics of thin disk with viscoelastic damping layer and the thin disk without viscoelastic damping layer disk are shown in Table 3.

By comparison of the data in Table 3, for the thin disk with viscoelastic material of Zn33, the relative errors of the results obtained from both testing and finite element method are relatively smaller since the (0,3) order, and the finite element method used in this paper is rational for the high order vibration. By comparison of the date between the disk and the thin disk with viscoelastic free damping layer of Zn33,the results indicated that the natural characteristics of the thin disk with viscoelastic free damping layer is changed, the natural frequency reduces at the low-order, the natural frequency of high-order such as the (0,5) and (1,2) order are increased.

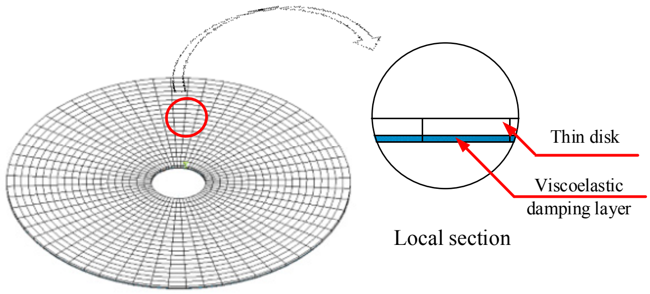

Fig. 3. Finite element model of thin disk with viscoelastic free damping layer

Tab. 3. The influence of viscoelastic damping layer on the natural frequency of thin disk

(*m*, *n*)	VE-disk /Hz		basis disk /Hz	
	Experimental	ANSYS	Experimental	ANSYS
(0,1)	50.00	58.86	50.01	61.819
(0,2)	78.13	83.88	84.38	88.091
(0,3)	175.00	175.28	181.25	184.073
(0,4)	303.13	309.99	315.63	325.537
(1,0)	356.25	403.79	403.13	424.038
(1,1)	462.50	435.50	none	457.342
(0,5)	496.88	483.39	487.50	507.635
(1,2)	650.00	545.83	540.63	573.202
(0,6)	721.88	698.11	678.13	733.126

3.3 The influence of viscoelastic damping layer on the traveling wave of rotating thin disk

The influence of viscoelastic damping layer on the traveling wave of rotating thin disk is plotted in Fig. 4, in which the rotating speeds various from 0 to 800 rpm. On the basis of static state analysis, rotation speed is added for z axis to analyze the rotation effect in the finite element method, considering the influence of pre-stress. Compared with the traveling wave curve of thin disk, it is found that the natural frequency of the thin disk with viscoelastic free damping layer change along with rotation speed, due to the similar "centrifugal stiffening effect" produced by the stiffness of viscoelastic material. The natural frequency of forward traveling wave of each order increase gradually with the rotating speed while the natural frequency of backward traveling wave has decreased at first and then increase. Considering the action of centrifugal force may lead to stiffening effect and the effect of vibration reduction on viscoelastic material the of thin disk with different rotational speeds may be different, by comparison, it is obviously that the influence of vibration of the (0,1) mode is the significant. The effect of rotational speed on viscoelastic material is consistent with metal material, and the stiffening effect caused by centrifugal force is existed.

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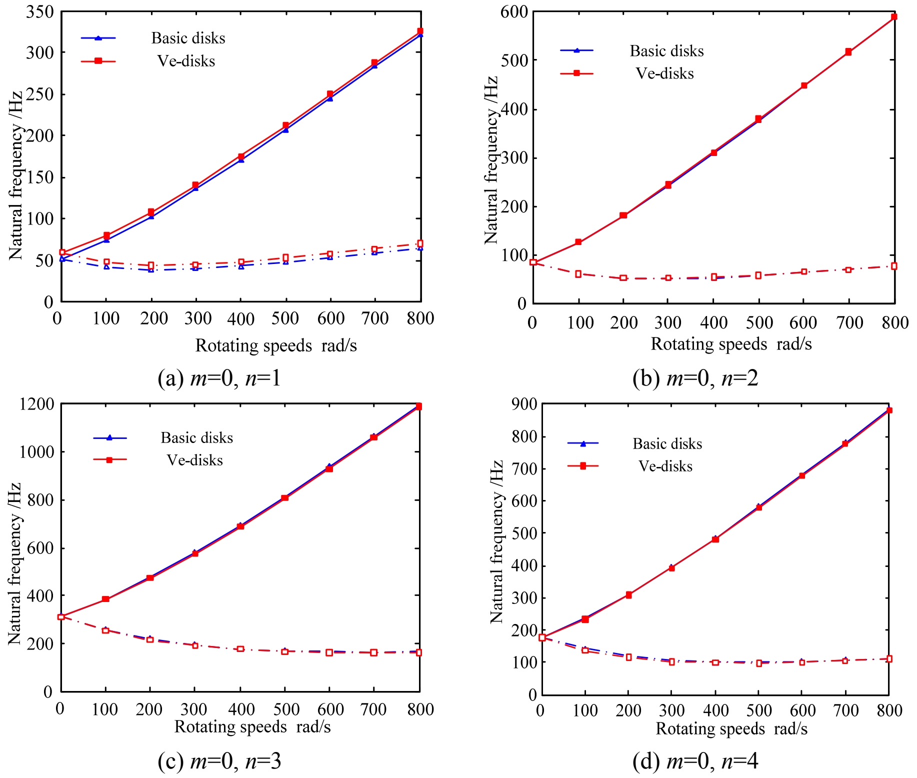

Fig. 4. The influence of rotating speed on natural frequencies of thin disk with viscoelastic free damping layer

4 Conclusion

In this paper, the dynamic characteristics of the rotating disk with the viscoelastic damping demand are analyzed, combining experimental tests with the finite element simulation, travelling wave characteristics of viscoelastic damping disk are investigated.

(1) The finite element method is introduced to analyze the influence of rotating dynamic centrifugal stiffening effect on the natural characteristics of the disk with viscoelastic damping layer treatment.

(2) The influence of viscoelastic materials on natural frequencies of thin disk with the different vibration orders are compared by vibration curves of traveling wave. By coating viscoelastic free damping layer, natural frequency of the low order reduced, and the high order increased. It can be found that, in rotating condition, the similar "centrifugal stiffening effect" will occur on the stiffness of viscoelastic material with the rotational speed, the natural frequency of the disk with viscoelastic free damping layer will change along with the rotation speed.

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