

# Parameter identification of an ARX type ship motion model using system identification techniques

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**Abstract:** The paper concerns parameter identification of an ARX type ship motion model for a real ship under the frame of system identification (SI) principles. Several groups of ship maneuvering experiments have been implemented to collect research data. An ARX model is taken for analysis and specifically identification of three in the concerned model becomes the research topic. The minimum sum of squares due to error is set as the error criterion for parameter identification of the model. A complete approach scheme synthesizing the recursive least square (RLS) and linear decreasing inertia weight particle swarm optimization (LDIWPSO) to get the parameter identification result is proposed. Through the application case, the proposed strategy is verified feasible and successful: the ordinary RLS can give initial identification results from experiments, as well as, the LDIWPSO is able to find a final optimal identification result by minimizing the global SSE. The accuracy of the identified parameters is checked by comparing plant output and model output and analyzing the test of goodness of fit. A final recommended decision on parameter identification and the complete ARX model is achieved. The given strategy can be applied to parameter identification problems in engineering due to its feasibility, effectiveness, and convenience.

**Key words:** system identification, parameter identification, ARX ship motion model, recursive least square (RLS) algorithm, linear decreasing inertia weight particle swarm optimization (LDIWPSO) algorithm

## 1. Introduction

### 1.1 Research background

Ship motion mathematical model must be established to study the ship's motions and effectively control the ship <sup>[1]</sup>. Capability to create highly accurate mathematical models for adequate simulation of the manoeuvring motion of real vessels is of great practical value, primarily due to the ever increasing demand by numerous ship handling

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simulation centres, as well as by enterprises developing computer-based bridge simulators [2]. Sutulo and Soares [3] have discussed number of selected topics related to ship mathematical models used in ship manoeuvring, which is mainly for simulation purposes.

A ship motion model in general consists of two elements: the model structure and the parameters. Thus, there are mainly two parts of work need to be done to model a ship's motion: determining/selecting the model structure and identifying/estimating the parameters composing the model. In terms of model structure, so far there are four main kinds of types for choice: the integrating mathematical model, the separating mathematical model (a kind of model proposed by the Ship Manoeuvring Mathematical Model Group (MMG) of Japan Towing Tank Conference (JTTC), so the separating mathematical model is also called MMG model), the responding mathematical model and the difference equation type model. When a model structure (model type) is determined/selected, the remaining work is to estimate the parameters constructing the model.

At present, the theoretical systems of ship motion modeling based on the four kinds of modeling mechanisms are very complete, and the model structure adopted in each kind of modeling methods is relatively fixed or relatively easy to determine. As a result, when modeling ship motion based on the existing structures of models, the main content or the difficulty of the work does not pointing to selecting/determining the model structure, but to identifying the parameters in the model-a parameter estimation process by using effective methods.

## 1.2 Review of related literatures

The concept of system identification (SI) was puts forward by Zadeh in 1962 for the first time, which means the determination, on the basis of observation of input and output, of a system within a specified class of systems to which the system under test is equivalent; determination of the initial or terminal state of the system under test [4]. Ljung presented that the identification procedure is based on three entities: the data, the set of models, and the criterion. Identification, then, is to select a model in the model set that describes the data best, according to the criterion [5]. Generally speaking, SI is a kind of modeling methods [6]. SI techniques are developed in control engineering to build mathematical models for dynamical systems [7], which can make or improve the mathematical model of a system based on the experimental data [8, 9].

SI techniques open new avenues to parameter identification of a ship motion model. Since 1970s, SI techniques have been successfully applied in the study of ship motion modeling. E.g. all parameters of the mathematical ship maneuvering motion model were able to be estimated according to the transfer function between single input (steering rudder) and multi-output (manipulation motion state) by using SI. Hwang [10] and Wang [11] introduced in detail that the SI technique was applied to model dynamic ship motion system. Up to the present, there have been many SI techniques employed to the parameter identification of ship motion models. Some examples are the ordinary least squares (LS) algorithm [12], improved LS algorithm (e.g. the constrained least square (CLS) method [7], etc.), the recursive least square (RLS) algorithm [13], the improved RLS algorithm (e.g. the lattice recursive least square (LRLS) algorithm [14],

etc.), maximum likelihood (ML) estimation method<sup>[15]</sup>, model reference method (MRA)<sup>[16, 17]</sup>, recursive prediction error (RPE) method<sup>[18, 19]</sup>, frequency spectrum analysis (FSA) method<sup>[20, 21]</sup>, kalman filter (KF) and the extend kalman filter (EKF)<sup>[7, 10, 22]</sup>, wavelet filtering technique<sup>[23]</sup>, particle swarm optimization (PSO) and the improved PSO algorithm<sup>[1, 24, 25]</sup>, genetic algorithm (GA)<sup>[2]</sup>, ant colony algorithm (ACA)<sup>[26]</sup>, fruit fly optimization algorithm (FOA)<sup>[27]</sup>, and support vector machines (SVM)<sup>[28, 29]</sup>, etc. Sutulo and Soares<sup>[3]</sup> have given a special attention to comparative evaluation of methods for estimating manoeuvring model's parameters including application of optimal experimental designs to captive-model tests and various SI techniques. With the development of experimental measurement technology today, SI shows its new vitality more and more in modeling the complex ship maneuvering motion system. Parameter identification of a concerned type of model can be realized by adopting SI technique.

### 1.3 What the paper focus on

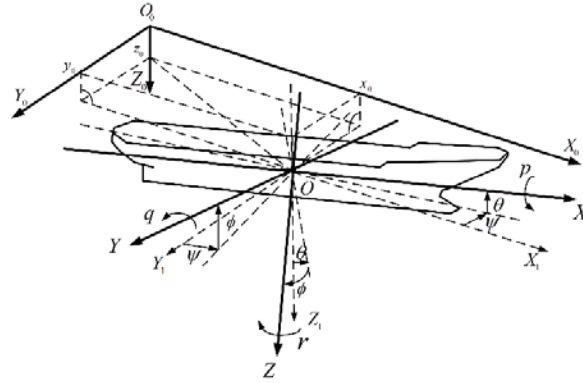
This paper mainly focus on parameter identification of an autoregressive exogenous (ARX) type ship motion model from perspective of SI. Following the guidance of the existing contributions, the core and main efforts are: (1) materials and method necessary to research goals will be prepared under the frame of the three entities: the data, the set of models, and the criterion. (2) due to the simplicity and effectiveness of the RLS algorithm and the adaptability of PSO, a complete feasible and effective approach scheme of parameter identification by employing RLS and PSO will be proposed and then verified through an application case. (3) the proposed approach scheme will be applied to parameter identification of an ARX model for a real ship to get a recommended result. Where in the application study, referring to Hwang<sup>[10]</sup>, the accuracy of final identification result will be checked by comparing the simulated motion (model output using identified parameters) and the ship trial record (the real plant output); then finally a recommended identification result and complete ARX model of the ship will be given.

## 2 Material and methods

The mentioned literatures concerned with parameter identification of ship motion model carry out their study under the three entities: the data, the set of models, and the criterion. Referring to those, the items in this section are arranged around the three entities. First, it gives the definitions of coordinate systems and variables for indicating ship motion (see 2.1). Second, it declares how the specific details of the three entities are prepared (see 2.2), e.g., as the foundation of research, the data collection and preprocessing based on ship maneuvering experiment (see 2.2.1), the selection of model class and determination of parameters (see 2.2.2), and the selection of error criterion for parameter identification (see 2.2.3): the minimum sum of squares due to error. Third, as a key to approach, the employed algorithms including RLS and PSO are introduced (see 2.3). Last, it gives a general approach scheme of parameter identification in this paper after the above explanation (see 2.4).

## 2.1 Settings of coordinate systems and variables

In the field of ship motion modeling, an earth fixed coordinate system and a body (the ship) fixed coordinate system, as well as the corresponding variables are usually used to describe the concerned ship's 6 degrees of freedom (6 DOF) of motion, as shown in Figure 1.



**Fig.1.** Coordinate systems and the corresponding variables for 6 DOF motion

Where, the frame  $O_0-X_0Y_0Z_0$  refers to the earth fixed coordinate system while the frame  $O-XYZ$  refers to the body fixed coordinate system.  $O_0$  is an selected initial point on the earth's surface and  $O_0X_0$ ,  $O_0Y_0$ ,  $O_0Z_0$  respectively point to north, east and the earth's core.  $O$  is the original point of the  $O-XYZ$  system (usually the ship's center of gravity is set as the origin point) and  $OX$ ,  $OY$ ,  $OZ$  respectively point to the bow, starboard and the keel of the ship. Convenient for description, the frame  $O-X_1Y_1Z_1$  is the translation of  $O_0-X_0Y_0Z_0$ : centered at  $O$ . Referring to  $O_0-X_0Y_0Z_0$ ,  $x_0$ ,  $y_0$ ,  $z_0$  respectively are the coordinates corresponding to the three axis;  $\psi$ ,  $\phi$ ,  $\theta$  respectively are heading angle (also known as course), rolling angle, pitching angle, which are called Euler angle. Referring to  $O-XYZ$ , the unwritten symbols  $u$ ,  $v$ ,  $w$  respectively are the ship's surge velocity along  $OX$ , sway velocity along  $OY$  and heave velocity along  $OZ$ ; The symbols  $p$ ,  $q$ ,  $r$  respectively are the ship's rolling rate around  $OX$ , pitching rate around  $OY$  and yaw rate around  $OZ$ . Specific meanings of the symbols also can be found in the contributions of Sutulo and Soares [2, 3] and the other listed references.

## 2.2 The three entities for parameter identification

### 2.2.1 Data collection and pre-processing based on ship maneuvering experiment

It is an indispensable basic work to obtain observational data of the plant through appropriate, practical physical experiment. As the foundation, several groups of ship maneuvering experiments have been executed to support the research.

#### 2.2.1.1 A general introduction to the concerned ship

The concerned ship is named “YI CHANG HUI FENG 9”, whose appearance and main hull details are present in Figure 2 and Table 1.



**Fig.2.** The concerned ship: YI CHANG HUI FENG 9

**Table 1.** Main hull details of the target ship

Attribute names	Attribute values	Attribute names	Attribute values
Name	YI CHANG HUI FENG 9	Identification NO.	CN20058425641
Ship type	Multi-purpose	Registration NO.	2005H4300399
Built date	2005.Oct.08	Limited navigation areas	Class A
LOA	76.80m	Ship length	73.71m
Waterline length (full)	76.41m	Ship width	13.6m
Moulded depth	4.4m	Maximum height	17.00m
Draft(empty)	0.789m	Draft(full)	3.8m
Displacement(empty)	559.7t	displacement(full)	3274.55t
Engine rated power	0.257kw/r/min × 2	Engine reduction ratio	4:1

#### 2.2.1.2 Experiment environment

The experiment environment includes: Date: 2016 Sept. 21; Place: Gulaobei channel, Yangtze River; Air temperature: 22 °C; Wind condition: nearly no; Visibility: 10 km;

Flow speed: about 1.46 m/s (the mean of drifting speed of an empty plastic bottle for 2 times).

### 2.2.1.3 Equipment and methods for data collection

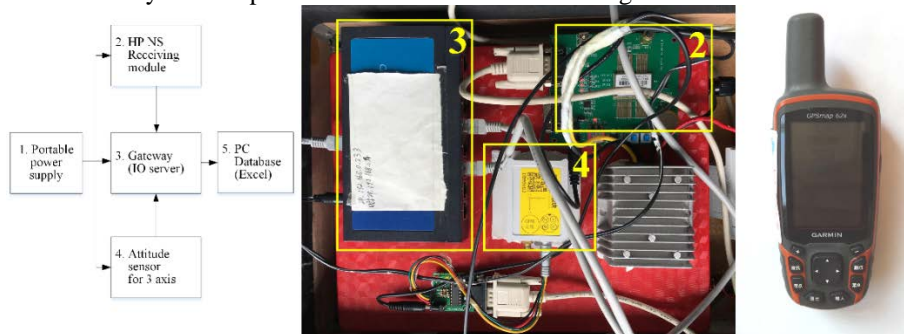
Data collection mainly refers to obtaining the input and output data of ship dynamic system. This section presents the equipments and methods used for data collection.

#### (1) System output: state variables record

System output refers to state variables of ship motion, such as the location and velocity in  $O_0-X_0Y_0Z_0$  frame, the attitude angle and angular velocity referring to  $O-XYZ$  frame, etc. A data collection system (DCS; primary version) is developed in advance for ship motion records. The system uses a high precision navigation satellite receiving module (HP NS receiving module) to get ship's location, as well as an attitude sensor inside an Inertial Navigation System (INS) to sense ship's attitude related to 3 axis. Where, the HP NS receiving module refers to a functional unit integrating the BeiDou Navigation Satellite System (BDS) / Global Positioning System (GPS) receiving modules. The collected information is transmit to a personal computer (PC) through a gate way (an IO server) and saved to the database (in the form of excel file).

At every sampling point, the DCS automatically records and gives system output in international standard unit, such as location and velocity in  $O_0-X_0Y_0Z_0$  frame, the attitude angle and angular velocity referring to  $O-XYZ$  frame, etc. There are 13~16 sampling points in each second and each sampling point is indicated by an ID number. Due to limited space, data format from the DCS is not presented here, a detailed show of data format can be referred in the supplement doc file if needed.

Besides, a handheld GARMIN GPS is used in the bridge to continuously record ship location automatically and occasionally mark the key positions manually. Thus, the tools for system output data collection are shown in Figure 3.



**Fig. 3.** Tools for system output data collection (left: conceptual structure of DCS; Mid: physical hardware connection of DCS; Right: The handheld GARMIN GPS)

#### (2) System input: control variable record

System input, mainly referring to rudder order and engine telegraph, is recorded manually on the bridge.

#### 2.2.1.4 Data pre-processing

It is need to process the collected raw data before it is used. For those collected by DCS, the missed data at a sampling point is added as well as the abnormal data is replaced both by using moving average method, according to equation (1).

$$y(k) = \frac{\sum_{i=1}^{i=5} y(k-i) + \sum_{i=1}^{i=5} y(k+i)}{10} \quad (1)$$

Where,  $k$  represents the sampling point serial number.

The mean value of the data at every sampling point within 1 second is used as the observation at this second, according to equation (2).

$$y(t) = \frac{\sum_{i=1}^n y_i(t)}{n} \quad (2)$$

Where,  $t$  represents the sampling time (s);  $n$  represents the number of samples completed by DCS within the  $t$ -th second;  $y_i(t)$  represents the observation at each sampling point,  $i=1,2,3,\dots,n$ .

#### 2.2.1.5 Data collection and display

Uploading the position data (recorded by GARMIN GPS) to Google Earth, ship track of the whole journey of experiment is shown in Figure 4 with the red solid line.



**Fig.4.** Ship track record by the handheld GARMIN GPS

During the whole journey, there are three periods when the system input (the rudder order) was relatively well observed and recorded. The three periods are respectively corresponding to Experiments 1~3, which are also shown in Figure 4. Thus, the system input-output data collected while carrying Experiments 1~3 are used for research.

For one thing, system output data collection while executing Experiments 1~3 are completed by the DCS automatically. For another, rudder order is got by manual record

on the bridge. Due to limited space, the source data in excel files is not presented here, only visualization of the acquired data is shown in the pictures. Part of system output collected by DCS is able to be provided if needed. After data collection and processing using the above mentioned ways, system input-output data are shown in Figure 5, where the red solid line refers to system output (observed ship course,  $\psi(0)$ ), while the blue solid line refers to system input (rudder order,  $\delta$ ). Considering rudder angle, note that the omitted “+” represents steering port side while “-” represents steering starboard side.

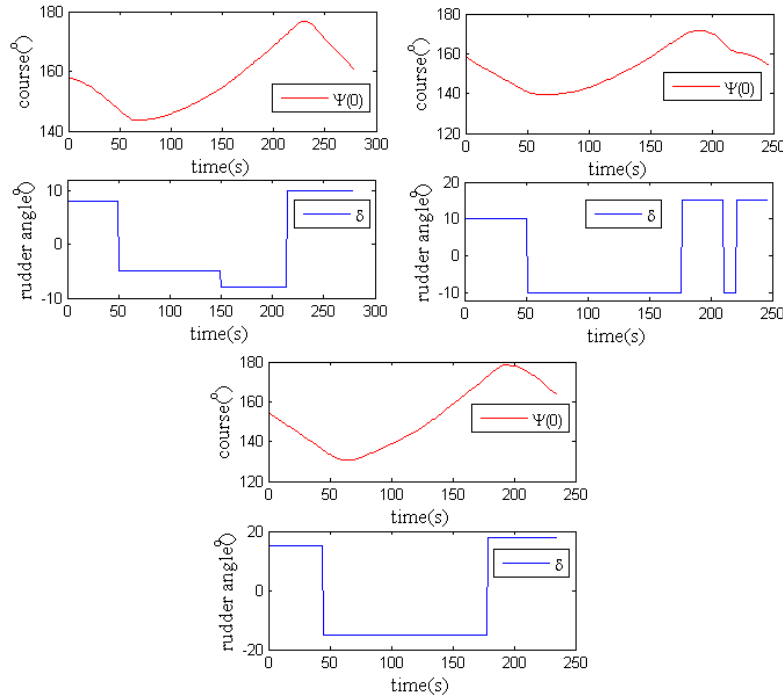


Fig. 5. System input-output data (Left: Experiment 1; Mid: Experiment 2; Right: Experiment 3)

## 2.2.2 Model class selection: the ARX type ship motion model

### 2.2.2.1 A general task description of ship motion modeling

According to the plant (the ship specifically) input  $\bar{\mathbf{X}}$  and the state output  $\bar{\mathbf{Y}}$ , the task of modeling ship motion from perspective of SI is to find out a model (or a function)  $M$ , which usually consist of two elements: the structure and parameters. What is to say, the model can be expressed by form (3).

$$M \{structure \quad parameters\} \quad (3)$$

Then, here comes the model output by the conceptual equation (4).

$$\hat{\mathbf{Y}} = M(\bar{\mathbf{X}}) \quad (4)$$

where,  $M$  is required to minimize some kind of tracking error. For instance, when taking same input  $\bar{\mathbf{X}}$  into account, it is required that the model  $M$  can minimize  $F(\bar{\mathbf{Y}} - \hat{\mathbf{Y}})$ , where  $F$  is a chosen function based on the actual demand.



### 2.2.2.2 Concerned model type and the parameters: the ARX type

Mainly analyzing the planar motion (3 DOF motion), the responding ship motion model captures chain reaction of rudder angle-turning speed-course ( $\delta \rightarrow \dot{\psi} \rightarrow \psi$ ) [30], which mainly describes ship course's response to the steering rudder in the plane. The paper goes from analyzing a first order linear responding ship motion model, which is expressed by equation (5).

$$T\dot{r} + r = K\delta . \quad (5)$$

Where in equation (5),  $K$ ,  $T$  represent ship maneuverability indices;  $r$  represents the angular velocity of turning ( $r = \dot{\psi}$  and  $\psi$  represents the ship course);  $\delta$  represents the current actual rudder angle.

Equation (5) is of a continuous form. However in engineering, a discrete form of the object's input and output is usually easy to be observed. That is to say, system output  $y(k)$  and input  $u(k)$  are easy to be obtained by the sensors,  $k$  represents the observation moment. Specifically, plant output  $\psi(k)$  and input  $\delta(k)$  are usually easy to get. This article is just about parameter identification of a specific model after getting plant input  $\delta(k)$  and output  $\psi(k)$ .

Set fixed sampling interval 1 second and discretize the continuous variables in equation (5) by using forward difference method, as shown by equations (6) and (7).

$$r(k) = \psi(k+1) - \psi(k) \quad (6)$$

$$\begin{aligned} \dot{r}(k) &= r(k+1) - r(k) \\ &= (\psi(k+2) - \psi(k+1)) - (\psi(k+1) - \psi(k)) \\ &= \psi(k+2) - 2\psi(k+1) + \psi(k) \end{aligned} \quad (7)$$

Then equation (5) can be discretized into equation (8).

$$T(\psi(k+2) - 2\psi(k+1) + \psi(k)) + (\psi(k+1) - \psi(k)) = K\delta(k) \quad (8)$$

Equation (8) can be rearranged to equation (9).

$$\psi(k+2) + \frac{-2T+1}{T}\psi(k+1) + \frac{T-1}{T}\psi(k) = \frac{K}{T}\delta(k) \quad (9)$$

Let  $a = \frac{-2T+1}{T}$ ,  $b = \frac{T-1}{T}$  and  $c = \frac{K}{T}$ , equation (9) can be concisely represented

by equation (10).

$$\psi(k+2) + a\psi(k+1) + b\psi(k) = c\delta(k) \quad (10)$$

Rewrite the equation (10) to standard ARX type, expressed by equation (11).

$$\psi(k) + a\psi(k-1) + b\psi(k-2) = c\delta(k-2) \quad (11)$$

So far, equation (11) is selected as the concerned CAR type model in this paper. Where, the parameters considered to be identified are  $a$ ,  $b$  and  $c$ .

An implicit form of equation (11) is equation (12).

$$\psi(k) = \phi(k) \cdot \mathbf{p} \quad (12)$$

Where,  $\phi(k) = [\psi(k-1) \quad \psi(k-2) \quad \delta(k-2)]$ , and  $\mathbf{p} = \begin{bmatrix} -a \\ -b \\ c \end{bmatrix}$ .

Then, the parameter vector to be identified is  $\mathbf{p}$ . For distinction, the symbol  $\hat{\mathbf{p}}$  is utilized as the estimation of  $\mathbf{p}$ .

### 2.2.3 Error criterion: minimizing sum of squares due to error

The specific mathematic model of tracking error determines the quality of identification result. Usually, a function of tracking error is used as the criterion to judge the parameter identification quality. The smaller the function value is, the higher the precision is. Generally for a single input single output (SISO) model, the error criterion can be implicitly expressed by equation (13).

$$\min J = f(e(k)) \quad (13)$$

Where,  $J$  refers to the evaluation index (function value),  $f$  refers to some specific function,  $e(k)$  refers to the tracking error at the  $k$  step,  $k$  refers to sampling time. For a SISO model, the tracking error  $e(k)$  is popularly calculated by (14).

$$e(k) = y_M(k) - y_P(k) \quad (14)$$

Where,  $\psi_M$  represents the model output by using the identified parameters,  $\psi_P$  represents the plant output.

Specifically for our topic, since the concerned ARX model employs single input (rudder angle  $\delta$ ) and gives a single output (ship course  $\psi$ ), the concepts of equations (13) and (14) can be used. Thus, based on the minimum sum of squares due to error (SSE), the following equation (15) is introduced as the criterion in the paper.

$$\min J_{SSE} = \sum_{m=1}^K J_{SSE}^m \quad (15)$$

Where,  $J_{SSE}^m$  is calculated by using equation (16).

$$J_{SSE}^m = \sum_{t=1}^{N_m} e_{\psi}(t)^2 \quad (16)$$

And,  $e_{\psi}(t)$  is calculated by using equation (17).

$$e(k) = \psi_M(k) - \psi_P(k) \quad (17)$$

In equations (15) ~ (17),  $k$  refers to sampling time;  $\psi_M$  represents the ARX model output;  $\psi_P$  represents the ship output;  $e(k)$  refers to the tracking error at the  $k$  step;  $m=1,2,\dots,K$  represent the  $m$ -th experiment;  $N_m$  is the total number of sampling points in the  $m$ -th experiment; then  $J_{SSE}^m$  represents the SSE corresponding to the  $m$ -th experiment;  $K$  is the total number of experiments ( $K=3$  since there are 3 groups of experiments); then as evaluation index, the final  $J_{SSE}$  refers to a global SSE.

The error criterion in this paper, equation (15), aims at minimizing a global SSE so that the final achieved ARX model can hold a universal adaptability: suitable for fitting output data from any group of experiment.

## 2.3 Employed algorithms and settings

Suitable algorithms ensures the feasibility and efficiency of solving the problem-parameter identification. The traditional methods, or recently new born optimization algorithms can be introduced to search a best solution. Given the simple principle and convenient implementation of least squares estimation (LSE) algorithms, the paper respectively applies the recursive least square (RLS) algorithm to estimate parameters from each experiment, in order to achieve an initial identification result. Afterward, one of the swarm intelligence optimization algorithms (SIOAs), the linear decreasing inertia weight particle swarm optimization (LDIWPSO) will be introduced to search a final identification result, which can minimizing the global SSE.

### 2.3.1 The recursive least square (RLS) algorithm

#### 2.3.1.1 Algorithm description

Generally, consider the follow universal ARX equation (18), which is designed to model a plant:

$$A(z^{-1})y_M(k) = B(z^{-1})u_M(k - nu_M) \quad (18)$$

where,  $k$  represents current time;  $y_M(k)$  is model output;  $u_M(k)$  is model input;  $A(z^{-1}) = 1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + \dots + a_n z^{-na}$ ;  $B(z^{-1}) = b_1 + b_2z^{-1} + b_3z^{-2} + \dots + b_{nb}z^{-(nb-1)}$ ;  $z^{-1}$  is the backward shift operator;  $na$  and  $nb$  respectively denote the length of coefficient vector  $A$  and  $B$ ;  $nu_M$  is the model input ( $u_M(k)$ ) lag.

If  $na$ ,  $nb$  and  $nu$  have been already determined, the next step is the parameter identification of  $A$  and  $B$ . Note that  $u_M(k)$  is set same with the plant input  $u(k)$  for parameter identification problem, which means  $u_M(k) = u(k)$ .

Convert (18) to an implicit form, referring to equation (19)

$$y_M(k) = \boldsymbol{\varphi}_M^T(k) \boldsymbol{\theta} \quad (19)$$

where,  $\boldsymbol{\varphi}_M^T(k)$  is called an information vector composed by  $y_M(k)$  and  $u_M(k)$ .

By contrasting (19) with (18),  $\boldsymbol{\varphi}_M^T(k)$  is indicated by equation (20).

$$\boldsymbol{\varphi}_M(k) = [y_M(k-1), y_M(k-2), \dots, y_M(k-na), u_M(k-nu_M), u_M(k-(nu_M+1)), \dots, u_M(k-(nu_M+nb-1))] \quad (20)$$

Now the newly introduced variable  $\boldsymbol{\theta}$ , where the original  $A$  and  $B$  are inside, becomes the parameter vector to be identified.

Then the RLS algorithm for parameters identification of equation (18) after getting system input-output is given by equation (21):

$$\begin{cases} \hat{\boldsymbol{\theta}}(k) = \hat{\boldsymbol{\theta}}(k-1) + \mathbf{P}(k) \boldsymbol{\varphi}(k) e(k) \\ e(k) = y(k) - \boldsymbol{\varphi}^T(k) \hat{\boldsymbol{\theta}}(k-1) \\ \mathbf{P}^{-1}(k) = \mathbf{P}^{-1}(k-1) + \boldsymbol{\varphi}(k) \boldsymbol{\varphi}^T(k) \end{cases} \quad (21)$$

where,  $\mathbf{P}$  is a covariance matrix;  $e(k)$  in the equation is called the innovation. Since it has only one dimension, the  $e(k)$  here is known as a scalar and is more accurately considered as single-innovation. Thus, the RLS algorithm mentioned here can also be called single innovation recursive least square (SIRLS) algorithm.

Concerning each experiment, equation (21) is adopted to estimate the parameters in equation (11) or (12), and, the identification results given by RLS is marked with  $\mathbf{p}_{\text{RLS}}$ .

### 2.3.1.2 Algorithm settings

The starting values when implementing the ordinary RLS are set as follows.

Starting value of the parameters:  $\mathbf{p}(0) = [a(0) \ b(0) \ c(0)] = [0 \ 0 \ 0]$

Starting value of the covariance matrix:  $\mathbf{P}(0) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot 10^3$

### 2.3.2 The linear decreasing inertia weight particle swarm optimization (LDIWPSO) algorithm

Particle swarm optimization (PSO) algorithm is a stochastic optimization method developed by Dr. Kennedy and Dr. Eberhart in 1995 [31, 32], who were inspired by social behavior of bird flocking or fish schooling. SHI etc. studied the basic characteristics of PSO [33], got some early experience on parameter selection in the algorithm. Then they presented a new concept about inertia weight to improve the original PSO, which aimed at providing a balance between global and local searching, and concluded that the PSO with the fixed inertia weight in the range [0.9, 1.2] on average will have a better performance [34]. Furthermore, researchers have introduced variation of inertia weight to improve performance of PSO, such as the linearly decreasing inertia weight [35, 36], sigmoid decreasing inertia weight [37], etc. To distinguish the varied forms of PSO, the paper here would like to call the original one developed by Dr. Kennedy and Dr. Eberhart in 1995 the basic PSO, and call the inertia weight modified algorithm proposed by SHI etc. in 1998 the standard PSO. Since a linear decreasing inertia weight PSO (LDIWPSO) is employed in this paper, the introduction of LDIWPSO and the setting of the algorithm is given below.

#### 2.3.2.1 Algorithm description

The mathematical model of LDIWPSO in the case of optimizing multi-dimension function (the objective function) is as follows.

Objective function:  $f(x_1 \ x_2 \ \dots \ x_D)$ , where  $D$  is the dimension of the search space.

Position of a particle:  $x_i = (x_{i1}, x_{i2}, \dots, x_{iD})$ , where  $i = 1, 2, \dots, NP$  and  $NP$  represents the total number particles.

Velocity of a particle:  $v_i = (v_{i1}, v_{i2}, \dots, v_{iD})$ , where  $v_{id} \in [v_{\min}, v_{\max}]$ ,  $v_{\min}$  and  $v_{\max}$  are assigned boundary velocity.

Personal best:  $p_{best} = (p_{i1}, p_{i2}, \dots, p_{iD})$ , where  $p_{best}$  refers to the best position (a solution to the objective function) searched by the  $i$ -th particle so far.

Global best:  $g_{best} = (p_{g1}, p_{g2}, \dots, p_{gD})$ , where  $g_{best}$  refers to the best position (the best solution to the objective function) searched by the whole particles so far.

When  $p_{best}$  and  $g_{best}$  obtained at current iteration  $k$ , the velocity and position of a particle are updated according to equations (22) and (23).

$$v_{id} = w * v_{id} + c_1 r_1 (p_{id} - x_{id}) + c_2 r_2 (p_{gd} - x_{id}) \quad (22)$$

$$x_{id} = x_{id} + v_{id} \quad (23)$$

Where:  $d \in \{1, 2, \dots, D\}$ ;  $c_1$  and  $c_2$  represent the learning factors (also called the acceleration constants);  $r_1$  and  $r_2$  are uniformly distributed random numbers,  $r_1, r_2 \in [0, 1]$ .  $w$  represents the inertia weight,  $w \in [0, 1]$ .

The linearly decreasing inertia weight ( $w$ ) changes according to equation (24).

$$w_k = w_{\max} - k \frac{w_{\max} - w_{\min}}{MC} \quad (24)$$

Where:  $w_k$  is the variable weight applied to (22);  $w_{\max}$  and  $w_{\min}$  are limits on inertia weight;  $MC$  represents the maximum iteration cycle.

When the maximum number of iteration reached, output the  $g_{best}$  (the best position of particles) as the global best solution to the objective function.

### 2.3.2.3 LDIWPSO's adaptability to parameter identification

LDIWPSO's adaptability to parameter identification is presented as follows.

#### (1) The SSE based fitness function and fitness degree

Based on the requirement of error criterion, the smaller  $J_{SSE}$  is, the higher the accuracy of parameter identification is, which means that the estimated parameter is closer to real value. Thus, a reciprocal of  $J_{SSE}$ ,  $\frac{1}{J_{SSE}}$  can be regarded as a fitness function by which the fitness degree ( $d_{Fitness}$ ) of a particle is calculated. What is to say, the equation (25) can be used to evaluate particle' positions (the quality of parameter identification).

$$d_{Fitness} = \frac{1}{J_{SSE}} \quad (25)$$

#### (2) The corresponding relation analysis

The corresponding relation among particle swarm behavior, multi-dimension function optimization and parameter identification of the ARX model is listed in Table 2.

**Table 2.** The corresponding relation: LDIWPSO's adaptability

Particle swarm behavior	Multi-dimension function optimization	Parameter identification
All of the possible locations	Feasible solutions	Various values of the parameters $a, b, c$
Particle's fitness degree	Quality of the feasible solution	Performance index ( $J_{SSE}$ ) value in accordance with different $a, b, c$
The speed of finding the optimal location	Speed of optimizing feasible solutions	Speed of optimizing the performance index $J_{SSE}$
The optimal location corresponding to maximum fitness degree	The optimal solution	The optimal $a, b, c$ minimizing the performance index $J_{SSE}$

Thus, it can be seen that parameter identification of the ARX model is able to be inverted as a nonlinear function optimization problem, which could be solved using LDIWPSO. Similarly, the LDIWPSO's identification result is symbolled with  $\hat{\mathbf{p}}_{LDIWPSO}$ .

### 2.3.2.3 Related settings for LDIWPSO searching process

Related settings for executing LDIWPSO to search a most suitable result are given as follow.

Dimension of the search space:  $D=3$

The searching space: upper bound:  $\mathbf{p}_{ub} = \max \hat{\mathbf{p}}_{RLS}$ ; lower bound:  $\mathbf{p}_{lb} = \min \hat{\mathbf{p}}_{RLS}$ . Where,  $\hat{\mathbf{p}}_{RLS}$  represents the initial identification results by executing RLS.

Population size (Number of particles):  $NP=80$

Maximum iteration cycle:  $MC = 20$

The flying speed: Maximum flying speed:  $V_{max} = 0.5$ ; Minimum flying speed:  $V_{min} = -0.5$

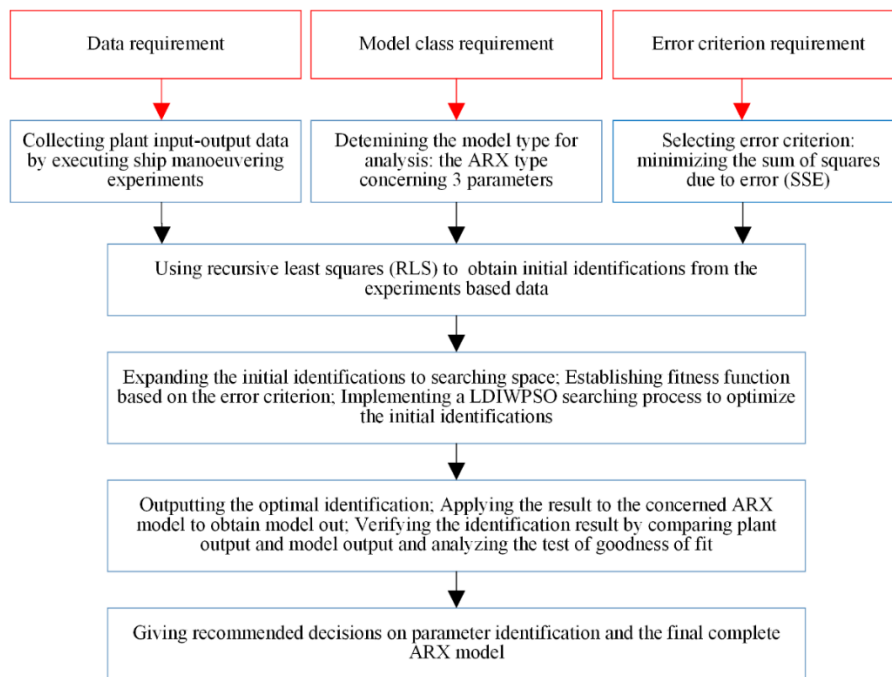
Learning factor:  $c1=1.3$  and  $c2=1.7$ .

Limits on inertia weight: Maximum inertia weight  $w_{max} = 0.9$ , Minimum inertia weight  $w_{min} = 0.1$ .

### 2.4 The approach scheme of parameter identification

Since the work goes under the frame of SI theory, the three entities (system input-output data of ship motion, concerned model class, and the tracking error criterion) for parameter identification should be meet first. The paper (1) collects plant input-output data by executing 3 groups of ship maneuvering experiments; (2) take an ARX type model for analysis, where there are 3 parameters of the concerned model need to be estimated; and (3) selects the LS as the error criterion for parameter identification. The RLS algorithm is applied to obtain an initial identification result from each group of experiment. By expanding the initial identifications to searching space and using error criterion based fitness function, a LDIWPSO searching process is implemented to

optimize the initial identifications to get a better identification result, which minimizes the sum of squares due to error (SSE). LDIWPSO's contribution is verified by comparing plant output and model output and analyzing the test of goodness of fit. Then, a final recommended decisions on parameter identification and the complete ARX model can be achieved. After the just explanation, the approach scheme of parameter identification in this paper is shown in Figure 6.



**Fig. 6.** The approach scheme of parameter identification

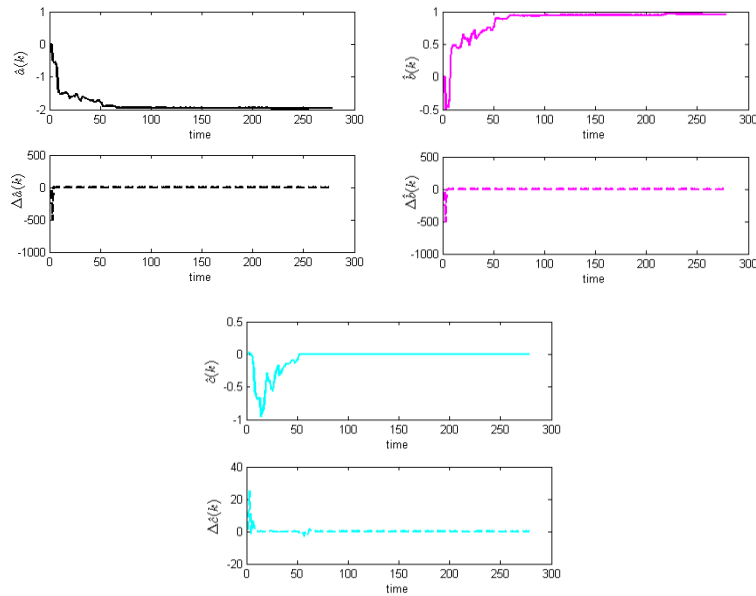
### 3 Results and discussion

This section goes in accordance with the steps describes in section 2.5 and Figure 6. Results and discussion after executing each step, including the algorithms, settings, etc., are given as follow.

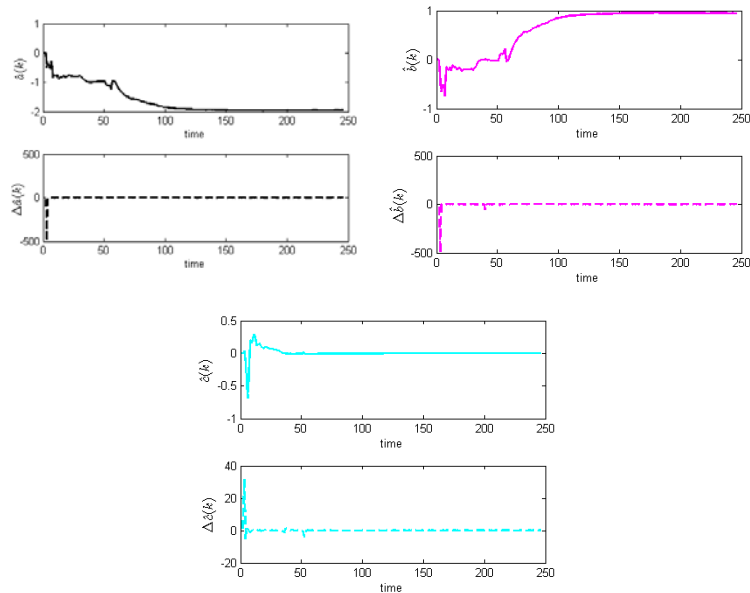
The three entities are prepared and illustrated first to meet the basic requirements. Three groups of ship manoeuvring experiments are implemented for data collection. Results of data collection and pre-processing can be seen in section 2.2.1. As explained in 2.2.2, the paper considers an ARX type model, where there are 3 parameters of the concerned model to be estimated. According to the error criterion, quality of the estimated parameters is designed to be judged by SSE. That is to say, the goal of parameter identification is to minimize the SSE, which is given in 2.2.3.

Based on the manoeuvring experiments, the RLS is used to identify the parameters

from each experiment, in order to give an initial identification. The initial identification processes and results by using RLS are given in Figures 7~9 and Table 2. Where, the symbol  $\hat{*}(k)$  represents the estimated value, and symbol  $\Delta\hat{*}(k) = \hat{*}(k+1) - \hat{*}(k)$  represents the change rate of  $\hat{*}(k)$ .

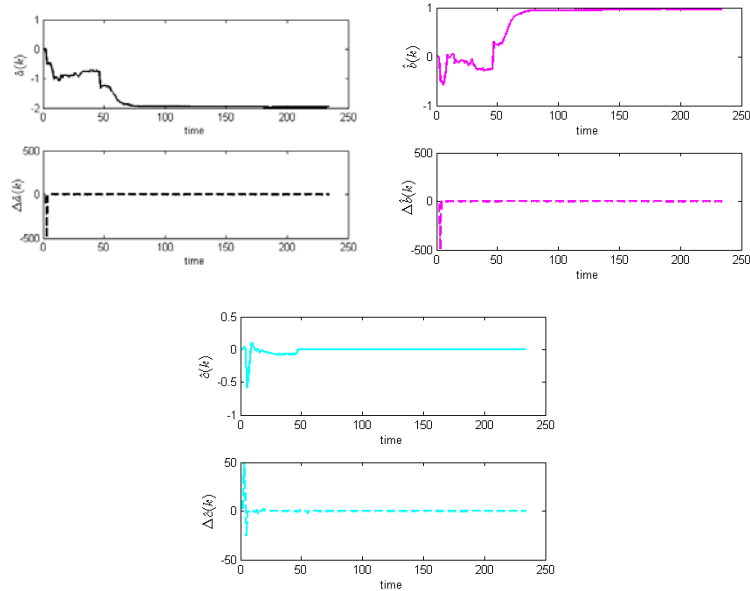


**Fig.7.** Parameter identification process from Experiment 1 (Left: *a* ; Mid: *b* ; Right: *c* )



**Fig. 8.** Parameter identification process from Experiment 2 (Left: *a* ; Mid: *b* ; Right: *c* )





**Fig. 9.** Parameter identification process from Experiment 3 (Left:  $a$  ; Mid:  $b$  ; Right:  $c$  )

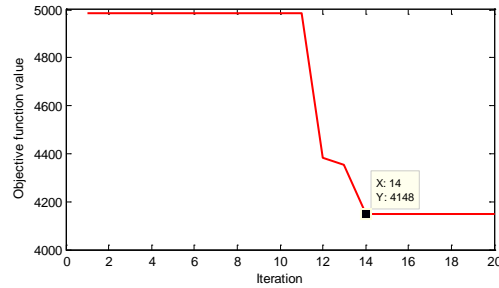
**Table 2.** Initial identification: estimated parameters from experiments by RLS

Initial identification		Group		
		Experiment 1	Experiment 2	Experiment 3
Parameter	$a$	-1.9553	-1.9460	-1.9596
	$b$	0.9553	0.9460	0.9596
	$c$	-0.0018	-0.0019	-0.0012

From Figures 7~9, the estimated value  $\hat{*}(k)$  is convergent: the change rate  $\Delta\hat{*}(k)$  gradually approaches to 0 and  $\hat{*}(k)$  tends to a fixed value. However, the initial identification results are not consistent: after RLS's iteration, the final estimated values obtained from the 3 experiments are different from each other, as shown in Figures 7~9 and Table 2. Besides, the result from an experiment can only minimize the SSE corresponding to the concerned experiment, and is not able to minimize the global SSE.

In view of this situation, the paper has planned to achieve a global optimal parameter identification of the AXR model by using LDIWPSO to minimize the SSE based objective function, which can be found in 2.2.3.

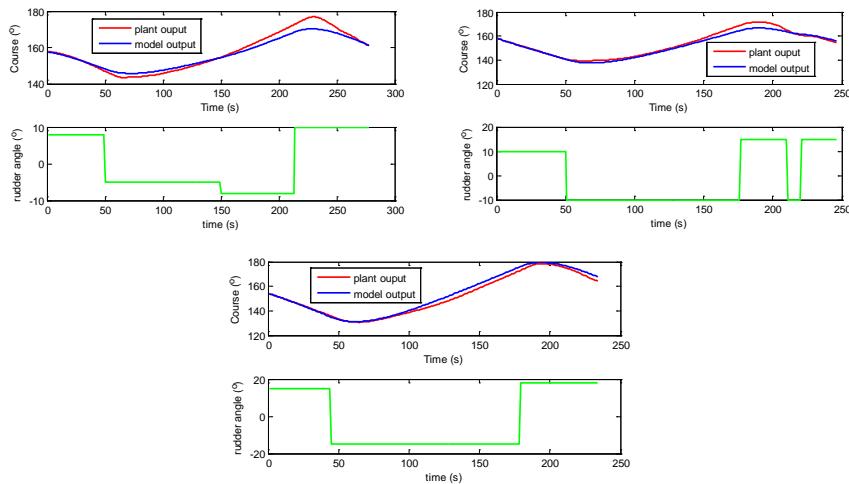
The automatic search process is carried out on MATLAB. After running the codes one time, the optimization process (change of the function value with iteration) by PSO is shown in Figure 10.



**Fig. 10.** LDIWPSO searching process: objective function value VS iteration

From figure 10, it can be seen that after 20 iterations, the minimum objective function value reaches 4148 at the 14<sup>th</sup> step, corresponding to an independent variable vector  $[-1.9596 \ 0.9596 \ -0.0013]$ . Thus, it is recognized that the identification result given by PSO is  $\hat{\mathbf{p}}_{\text{PSO}} = [a_{\text{PSO}} \ b_{\text{PSO}} \ c_{\text{PSO}}] = [-1.9596 \ 0.9596 \ -0.0013]$ .

Now comes testing the PSO based result. Similar with Hwang <sup>[10]</sup>, the accuracy of the obtained parameters by PSO is checked by comparing the simulated motion (model output using identified parameters) and the trial record (plant output: the real ship maneuvering trial record). Consistently, taking the above steering commands as input, the observed plant output (red line) and the model output (blue line) are shown in Figure 11 to make comparisons.



**Fig. 11.** Comparison between plant output and model output using recommended parameters

What is more, the test of goodness of fit indicated by the statistical indexes are listed in Table 3. Where, SSE: sum of squares due to error; SSR: sum of squares for regression; SST: sum of squares for total. CoD: coefficient of determination ( $R^2$ ).

**Table 4.** The test of goodness of fit

Index value		Category			
		Experiment 1	Experiment 2	Experiment 3	Final total
Statistical index	SSE	1792	1143	1213	4148
	SSR	18903	22993	65492	107388
	SST	20695	24137	66706	111537
	CoD	0.9134	0.9526	0.9818	0.9628

By applying the LDIWPSO's contribution, model output fits the plant output well, as shown in Figure 11. The final CoD ( $R^2$ ) reaches 0.9628, indicating that the identified parameters make the mathematical model high quality, as given in Table 3. Thus, the LDIWPSO searched results are treated as the final recommended parameters, and the complete ARX model is expressed by equation (26).

$$\psi(k) - 1.9596\psi(k-1) + 0.9596(k-2) = -0.0013\delta(k-2) \quad (26)$$

Thus, the propose approach scheme of identifying parameters of an ARX ship motion model is verified feasible and successful through the application case. It is thought that the given strategy can be applied to solve practical problems in engineering.

## 4 Conclusion

The paper concerns parameter identification of ARX model under the framework of SI's three entities: the data, the set of models, and the criterion. A complete technical route of estimating the parameters of the ARX model is proposed and applied to solving a practical problem. It is concluded after the thread of the research that:

(1) The approaches to the three entities (system input-output data of ship motion, concerned model class, and the tracking error criterion) for parameter identification are achievable, which provides the foundation of the research topic.

(2) The strategy of synthesizing the RLS and LDIWPSO to get the parameter identification result is verified feasible and successful through the application case. The ordinary RLS can give initial identification results from experiments; the classical LDIWPSO is able to find a final optimal identification result by minimizing the global SSE.

(3) It is inferred that the given strategy can be applied to parameter identification of other difference equation type motion model, e.g., the autoregressive (AR) type, the autoregressive moving average (ARMA) type, etc.

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