Optimal Design for Reinforcement Forms of Thin-walled Structure

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Abstract: In this paper, the stiffened structure of aircraft panel is taken as the research object to analysis its dynamic characteristics, based on the result, the optimization analysis are carried out to obtain reinforcement forms with the higher stiffness and reasonable modal distribution. The finite element models of thin-walled stiffened structures with different section shapes are established to study the effects of different section shapes and different numbers of stiffeners on the intrinsic properties of the thin panel structure. Subsequently, the influence of the reinforcement forms of different section shapes on the vibration characteristics and yield load of thin-walled structures is analyzed. Finally, taking the T-shaped reinforced panel structure as an example, an optimization analysis on its structural parameters of stiffeners was carried out to obtain reinforcement forms with the higher stiffness and reasonable modal distribution. The results indicated that different reinforcement forms have obvious influence on the intrinsic characteristics, vibration response and yield load. The maximum deformation and maximum stress value of the thin-walled structures of the optimized reinforcement forms can be significantly reduced. In practical applications, it is necessary to optimize the design to find the appropriate shape, the number and the structural parameters of the stiffeners.

Keywords: Thin-walled Stiffened Structure; T-shaped Stiffened Panel; Vibration Response Analysis; Structural Parameter Optimization Design

1 Introduction

In aerospace equipment, the thin-walled structures are largely used which are based on their advantages of smaller mass, less material and higher loads, taking the aircraft wall panels and skins, thin-walled engines of engines, the shell structure of the rockets or missiles as examples. Taking the aircraft as an example, the thin-walled structure is mostly composed of thin plates and stiffeners. The plates consist of skins, webs, partitions, etc. The stiffeners include purlins and beams, ribs and so on. The main function of the plate is to absorb the in-plane load and the vertical load, and the stiffener can bear most of the vertical load and to ensure that the plate has so much sufficient stability that to bear the in-plane load. Therefore, it has greatly engineering significance on study the influence of reinforced form and its optimal design on the bearing capacity of thin-walled structures.

In the study of the stability of thin-wall plate and shell structures, scholars have conducted a lot of research work through buckling theory, finite element method, experimental and statistical analysis of data. Based on the linear buckling theory and energy principle of classical plate shell, Chen et al have deduced the critical load calculation method for axial compression instability of reinforced thin-walled structures suitable for engineering applications[1]. Lancaster[2] analyzed the anomalous buckling behavior of thin-walled cylindrical shells and pointed out that the special boundary conditions, bottom support conditions and experimental assumptions in the experiment which has affected the experimental results. Shao et al[3] have pointed that the stability and carrying capacity of the thin-walled stiffened plates with three dimensions under the shear load by experiment and obtained buckling mode and buckling instability and failure load of the structure. The experiment shows that the composite stiffened plate still has a large bearing capacity after buckling instability, and the thickness of the skin and the number of the stiffeners also have a great influence on the buckling and failure load of the plate. He[4] performed a failure prediction on the axial compression failure of the composite stiffened cylindrical shell and used several failure criterion which are based on the degradation principle in order to analyze the proposed model. Sun et al[5] carried out experiments and numerical simulations on aero-reinforced aluminum plates with improved axial compression, the post-buckling analysis method was used to simulate the axial-stressed stiffened plates, and the simulation results were consistent with the experimental results. Li et al[6] studied the post-axial buckling analysis of an anisotropic cylindrical shell subjected to shear deformation, the results show that it is unstable between the post-buckling path of axial thin-walled cylindrical shell and the stiffened shell with the initial defects. Based on the stress characteristics of the thin-walled composite grid reinforcement structure, a new calculation method of the carvel built equivalent rigidity was deduced, it is considered the interactions between the stiffeners and panels, which were represented by a parametric representation of the grid cell structure layout[7]. A general mechanical analysis model is established, and the general linear characteristic equation for solving the buckling load of the grid stiffened plate is derived, which has a good application value for the optimization design of the reinforced structure[8].

At present, as for the optimization design of the reinforced thin-walled structure, especially for the shape of the shell structure and topology optimization of the stiffeners layout, many scholars have done a lot of work about it. Belblidia et al[9] used a hybrid algorithm to optimize the topological shape of the plate, and a suitable profile curve based on the topology optimization results, and then the size of the plate is optimized. The results show that this method can reduce the structural strain energy, maximize deformation and Mises Stress. Afonso[10] used a combination of topological optimization and size optimization to optimize the stiffened plates and shells, illustrating the effectiveness and flexibility of this method for stiffened plates and shells. Alinia[11] used ANSYS to study the optimization design of stiffened plates under the sheer loading, through theoretical calculations and examples of typical geometric panels, to verify the validity of the eigenvalue analysis for the first time. Banichuk et al[12] used genetic and neural network algorithms to study the multi-objective optimization problems of the geometric properties of thin-walled cylindrical shells. Iuspa[13] used the modified finite element method to establish the finite element model of simply-supported composite stiffened plates, and used a genetic algorithm to optimize the structural topology of stiffened plates. Su[14] used a genetic algorithm for node coding to solve the shape topology optimization and size optimization of the truss, the numerical experimental results show that the method has better convergence and robustness. Wang et al[15] used ant colony algorithm to optimize the design of composite stiffened plates. Buragohain[16] proposed a new method for fabricating composite stiffened cylindrical shells-filament winding, then preformed an experiment and a numerical simulation on the buckling of axially compressed cylindrical shells, cylindrical lattices and stiffened cylindrical shells. The results of the comparison show that the production method can improve the bearing capacity of the structure effectively. Bagheri et al[17] established an optimal design model based on genetic algorithm. The design variables include skin thickness, number of stiffeners and stiffener section size. Constraints include structural weight, axial buckling force and hoop buckling force. The objective function is the maximum fundamental frequency and the smallest structural weight which are used for the multi-objective optimization design of the hoop-stiffened cylindrical shell. Sun[18] applied topological optimization technology to study the effect of the skin thickness and the reinforcement and the number of the periodic grids on the optimization performance of grid-reinforced structures. The research results have important directive meaning on grid configuration design and selection of grid-stiffened structures. Wang[19] proposed a two-step optimization strategy to carry out design studies on the optimization of the layout of stiffened members. By substantially arranging the reinforcement locations without substantially changing the structural weight, the stiffness of the structure can be significantly improved, and the overall carrion capacity of the structure is improved, subsequently, the section size of the reinforcement was adjusted slightly to meet the design requirements for the maximum deformation of the structure, and an example was used to verify the feasibility and effectiveness of the proposed optimization method.

In this paper, the stiffened structure of aircraft panel is taken as the research object to analysis its dynamic characteristics and obtain reinforced forms with the higher stiffness and reasonable modal distribution by optimization analysis. However, in the case of thin-walled stiffened structures, how to perform dynamic analysis and the comparison and optimization of stiffened structural forms, and considering the influence of stiffeners layout on its buckling bearing capacity, further studies are still needed.

2 Structure analysis and modeling

2.1 Stiffened panel structure with different reinforcement forms

The air intake of a large transport aircraft auxiliary power unit (APU) uses a light-weight wallboard structure. Due to the complex transonic flow in the inlet, interaction between shock wave and boundary layer, pressure disturbance and buzzing in combustion chamber cause unsteady excitation of the inlet wall structure, which causes significant deformation and vibration of wallboard structure, and affects the flow performance of the APU inlet, resulting in fatigue failure of the inlet wallboard. Fig. 1 shows the structure of an airborne wallboard.

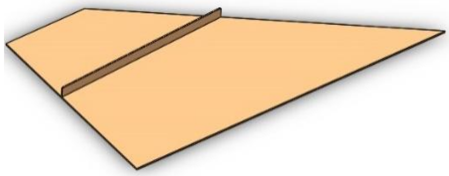


Fig. 1. The aircraft panel structure

In order to control large vibration of the APU intake wallboard and avoid cracks, it is currently planned to use a stiffened treatment scheme for the wallboard in the engineering design. Traditional connections between wallboard and stiffened structures include riveting, welding, bonding, etc. The combined wallboard has some problems of large assembly workload, uneven distribution of strength and stiffness, etc. At present, the stiffened integral wallboard structure is widely used. It is a combination of skin and stiffeners, made of the same material. The failure mechanism of the cracks in the APU inlet wallboard of a transport aircraft during the flight is analyzed. It is considered that the vibration and the resonance of the structure in the flight process lead to the vibration fatigue failure. Due to the severe restrictions on the weight of the aircraft, the existing design methods proposed to increase the numbers and thickness of the wallboard are lack of economy and advancement. Therefore, we expect to achieve the desired results via the dynamic analysis and optimization of the stiffener's form.

Four different types of stiffener are used, and the cross-sectional shapes are L-shaped, cap-shaped, Z-shaped, and T-shaped, respectively. The specific size parameters are shown in Fig. 2.

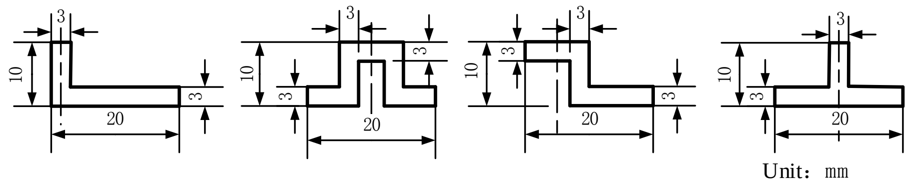


Fig. 2. Stiffener shape and sectional dimension

2.2 Finite element model of stiffened panel

Taking the T-shaped wallboard structure as an example, the finite element model of its stiffened wallboard is established. The selected study object is TC4 thin plate with reinforced section. The thickness of the wallboard is 3mm, the length is 600mm, the width is 400mm, and four T-shaped stiffeners are distributed on the wallboard with a spacing of 80 mm. The material parameters are shown in Table 1.

Table 1. Material parameters of stiffened panel structure

Elastic modulus E / Pa	Poisson's ratio μ	Density ρ /(kg / m^3)
1.138×10^{11}	0.3	4420

ISSN 2572-4975 (Print), 2572-4991 (Online)

The Solid186 unit is used to establish the finite element model of the cantilever board, and a fixed constraint is imposed on the head face of the wallboard along the stiffener direction to mesh the model. The model is meshed as the thin plates with T-shaped stiffeners having a total of 1590 units and 11080 nodes. The model is shown as Fig. 3.

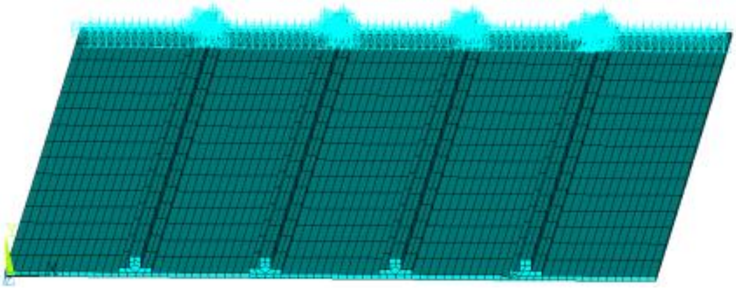


Fig. 3. FE model of cantilever stiffened panel

3 Effect of different reinforcement forms on the performance of thin plates

Vibration response and fatigue life of the structure are related to damping parameters and inherent characteristics of system. Therefore, the characteristic parameters such as natural frequency, damping ratio and vibration amplitude are selected as the indexes to evaluate the damping efficiency of damping materials.

3.1 Effect of different section shapes on the intrinsic characteristics of thin plates

The modal analysis of the above four models is carried out, respectively, and the first 8-order natural frequencies of the four cantilever plates are obtained as shown in Table 2.

Table 2. The first 8-order natural frequencies of plates with 4 reinforced forms (Hz)

Order	L-shaped	Cap-shaped	Z-shaped	T-shaped
1	17.409	23.808	26.196	17.509
2	33.735	50.217	38.490	35.948
3	102.46	132.11	116.84	103.85
4	121.39	148.02	148.18	125.11
5	124.13	175.63	154.48	129.65
6	208.98	277.78	228.58	218.89
7	262.40	315.57	287.77	276.03
8	275.85	355.03	309.74	277.08

Fig. 4 shows the effect of different stiffener sections on the natural frequency of thin plates. Comparing the calculation results of the natural frequencies, it can be found that the first 8-order natural frequencies of the stiffened plate with cap-shaped section are the highest, followed by the Z-shaped and the T-shaped stiffeners, and the first 8-order natural frequencies of the L-shaped stiffened plates are the lowest.

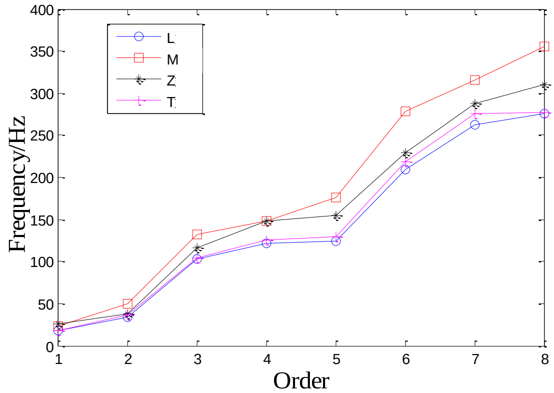
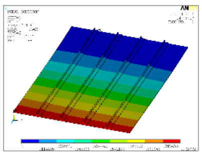
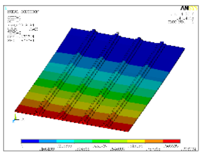
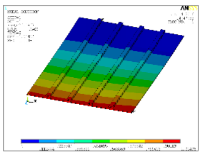
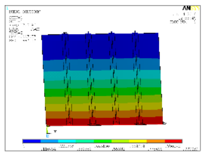
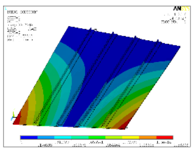
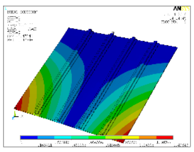
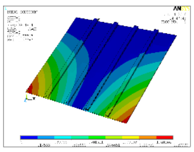
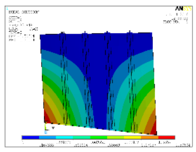
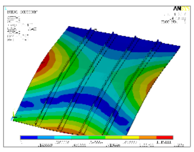
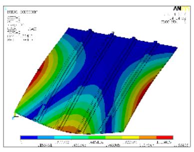
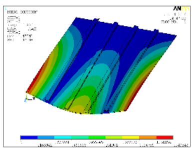
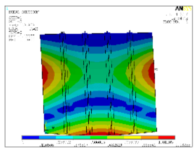
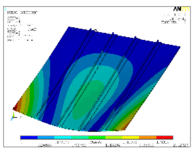
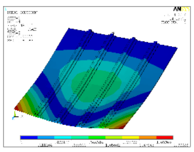
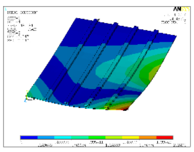
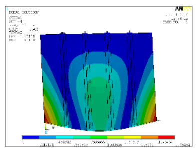


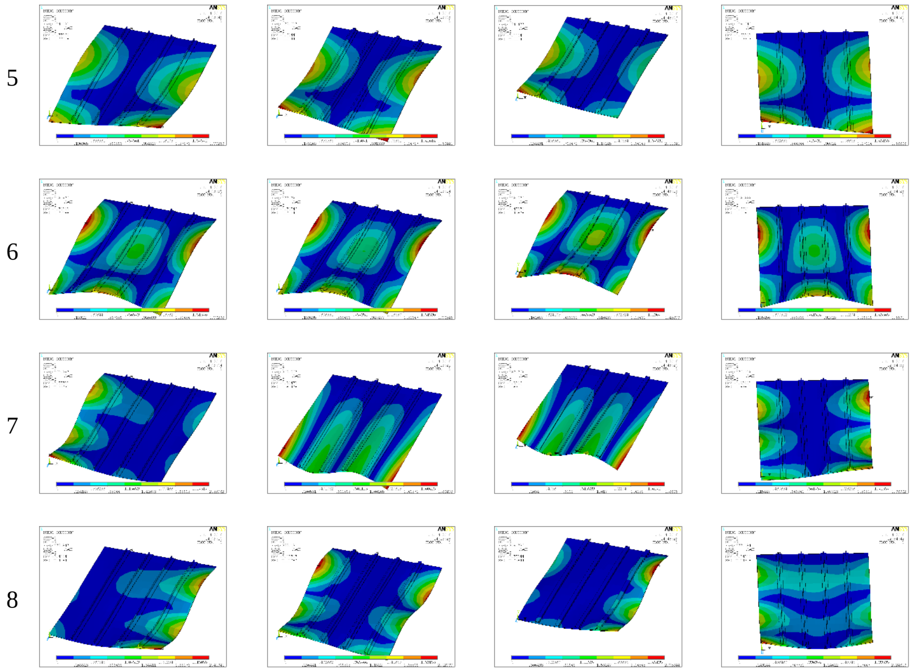
Fig. 4. The effect of different stiffener's sections on the natural frequency of thin plates

The modal analysis of the above four models are carried out, and the first 8-order modal shapes are obtained, as shown in Table 3.

Table 3. The first 8-order modal shapes of cantilever plates with four reinforcement forms

n	L-shaped	Cap-shaped	Z-shaped	T-shaped
1				
2				
3				
4				

ISSN 2572-4975 (Print), 2572-4991 (Online)



Comparing the first 8-order modal shapes of the stiffened plates with different cross-sectional shapes, it can be seen that from the third order, the cross-sectional shape of the stiffeners has a great influence on the modal shape of the thin plate, and the vibration form changes greatly. Among them, the modal shapes of L-shaped and T-shaped stiffened plates are more consistent, and the modal shapes of cap-shaped and Z-shaped stiffened plates are more consistent.

3.2 Effect of stiffener's number on the intrinsic characteristics of thin plates

Take stiffened plate with L-shaped stiffeners as a study object, and analyze the influence of the number of reinforced bars on the inherent characteristics of the thin plate. Set the number of reinforced stiffeners as 1 to 7, and obtain the first 8-order natural frequencies of the stiffened plate with different number of reinforced stiffeners, as shown in Table 4 and Fig.5.

Table 4. Natural frequencies of 7 cantilever plates with L-shaped stiffeners (/Hz)

Stiffener number	1^{st} order	2^{nd} order	3^{rd} order	4^{th} order	5^{th} order	6^{th} order	7^{th} order	8^{th} order
1	10.793	22.122	59.273	74.767	112.14	138.72	150.77	187.27
2	13.857	27.467	78.141	94.313	116.48	168.02	189.40	210.32

3	15.898	31.046	92.121	110.84	119.18	192.16	228.22	241.75
4	17.409	33.735	102.46	121.39	124.13	208.98	262.40	275.85
5	18.556	35.810	110.08	123.34	134.73	221.43	292.29	297.00
6	19.486	37.369	115.13	125.79	142.49	230.68	297.71	313.87
7	20.287	38.826	119.05	128.59	149.89	239.25	299.06	335.24

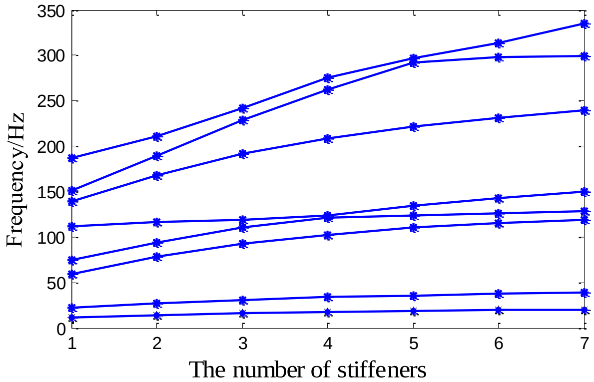


Fig. 5. Effect of number of stiffeners on the natural frequency of cantilever plate

From Table 3 and Fig. 5, it can be seen that with the number of stiffeners increasing, the natural frequency of each order gradually increases. Meanwhile, the mass of the thin-walled structure also increases, and so it is necessary to figure out the most suitable number of stiffeners via the optimum design according to actual requirements.

3.3 Effect of different reinforcement forms on vibration responses of thin plates

Apply basic incentives to finite element models of four stiffened plates, and use the modal superposition method, that is, calculate the response of the structure by multiplying the mode shape obtained by modal analysis by a factor and sum up, to process the response data of a node in the lower right corner of the cantilever end of the thin plate, the frequency response curves of four stiffened plates with different sections can be obtained, as shown in Table 5.

It can be seen from the response spectrum curves of the four stiffened plates in table 5 that vibration response of the first-order natural frequency of the reinforced plate T-shaped stiffeners under the base excitation is the largest, the vibration reduction of the reinforced plate with Z-shaped stiffeners is better, followed by the cap-shaped stiffeners.

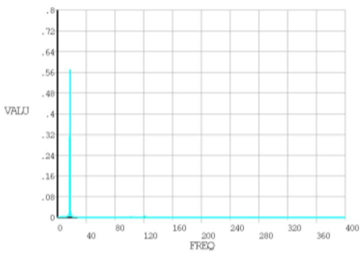
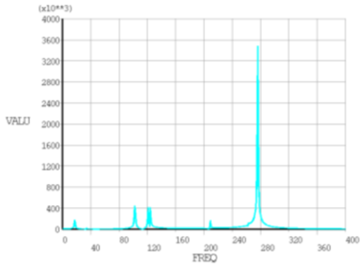
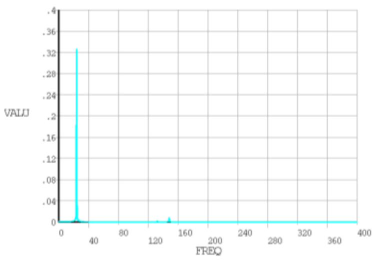
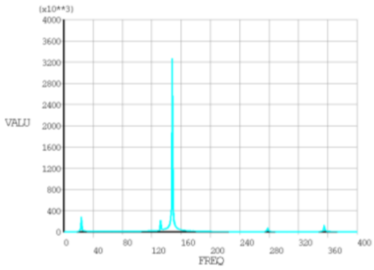
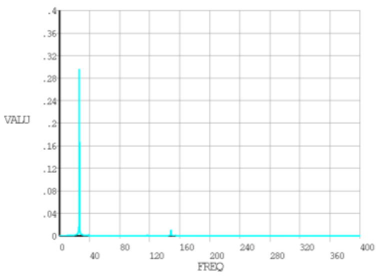
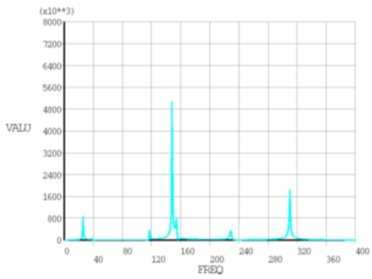
3.4 Effect of different reinforcement forms on buckling loads of thin plate

3.4.1 Pressure load condition

 ISSN 2572-4975 (Print), 2572-4991 (Online)

Apply a pressure of 1000N to the head face of four cantilever thin plates with different cross-sectional shapes, respectively, and perform static analysis, obtaining the results as shown in Table 6.

Table 5. Frequency response curves of selected nodes

Stiffener types	Displacement response spectrum curve	Stress response spectrum curve
L-shaped		
Cap-shaped		
Z-shaped		

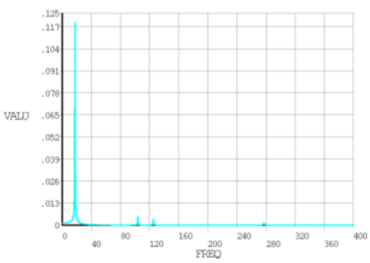
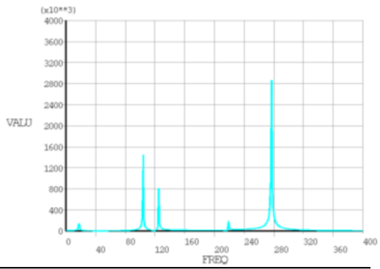
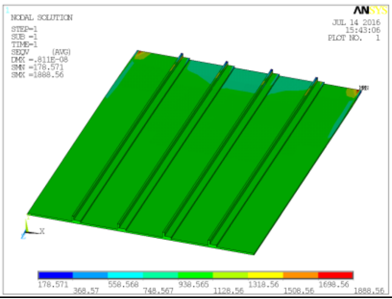
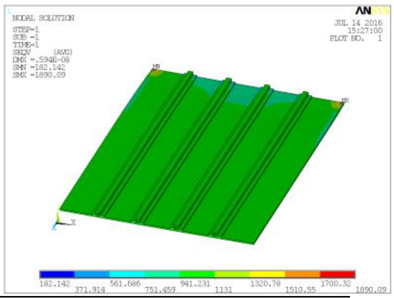
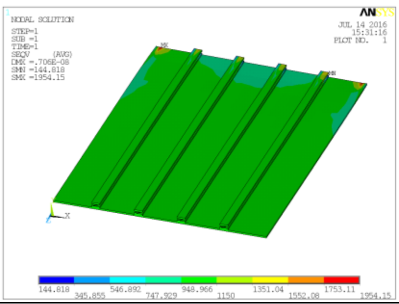
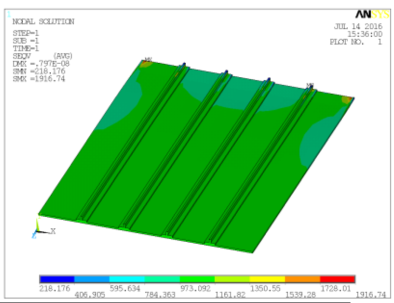
T-shaped		

Table 6. Load analysis of panels with different stiffener's shape

section form	L-shaped	Cap-shaped
maximum stress	1888.56	1890.09
stress distribu-tion		
section form	Z-shaped	T-shaped
maximum stress	1954.15	1916.74
stress distribu-tion		

From the above results, the maximum stress of the reinforced plates exists at the two corners of the fixed edge. The maximum stress of thin plates from the largest to the smallest are ones with Z-shaped, T-shaped, cap-shaped, L-shaped stiffeners.

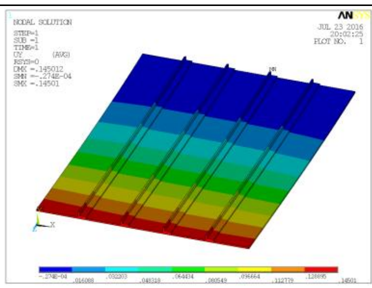
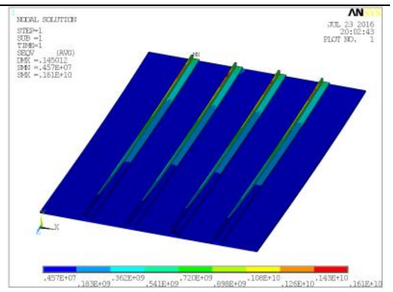
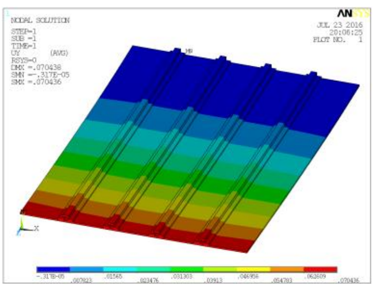
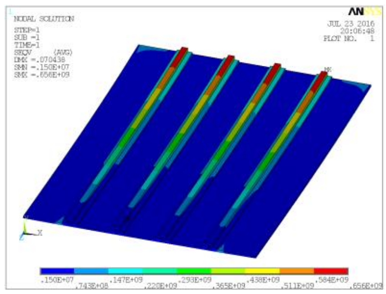
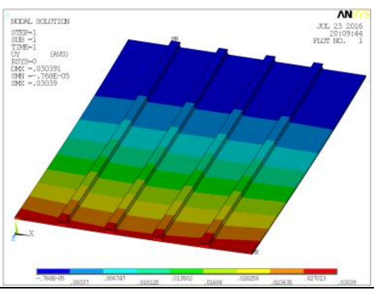
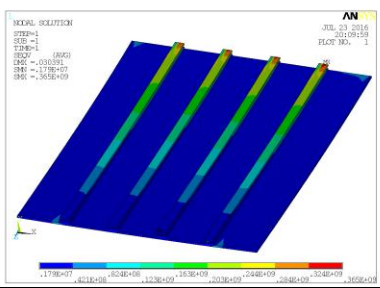
ISSN 2572-4975 (Print), 2572-4991 (Online)

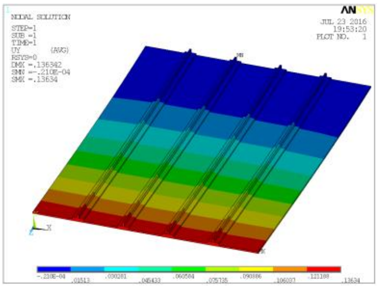
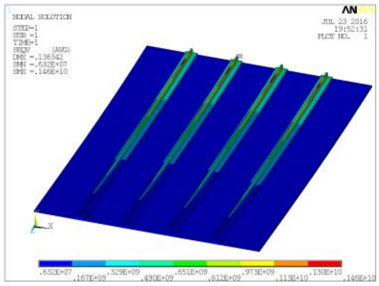
3.4.2 Line load condition

Apply line load to the cantilever end of the thin plate and apply 10N force to each node on the edge in the Y direction, the deformation and stress distribution of four different types of stiffened plates can be obtained by calculation, as shown in Table 7.

From table 7, it can be seen that the maximum stress of stiffened plates with different cross sections take occurs on the bars, when the cantilever ends are subjected to uniformly distributed line load. The maximum stress of thin plates from the largest to the smallest are ones with L-shaped, T-shaped, cap-shaped, Z-shaped stiffeners.

Table 7. Stress analysis of cantilever panels with different stiffener's shape

Stiffener types	deformation in Y direction	equivalent stress
L-shaped		
Cap-shaped		
Z-shaped		

T-shaped		

4 Optimization of stiffener section parameters of T-shaped stiffened panels

4.1 Preliminary analysis

Fig.6 is a cross-sectional view of the T-shaped stiffener, where H represents the height of the stiffener, T is the width of the stiffener tip, and S represents the ratio of width to height H/T.

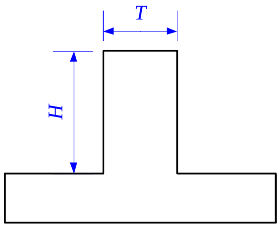


Fig. 6. Sectional view of stiffener

The initial conditions are set as follows: when the stiffener width to height ratio S=1, the width and height are equal and both are 0.007m, the maximum deformation of the stiffened plate in the Y direction is 0.66e-8m, and the maximum equivalent stress is 1918.18Pa, as shown in Fig. 7 and Fig. 8.

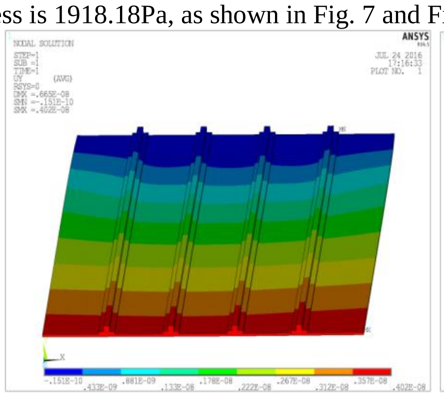


Fig. 7. Deformation under initial conditions

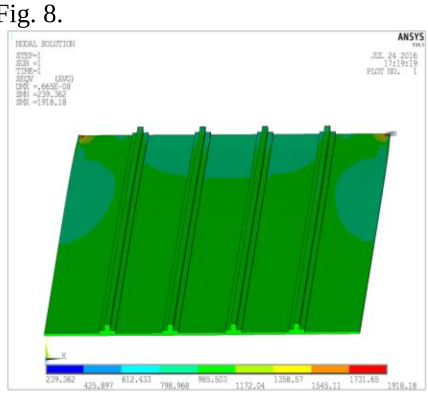


Fig. 8. Stress under initial conditions

ISSN 2572-4975 (Print), 2572-4991 (Online)

4.2 Solution of optimizing method

Select the height H and width T of the T-shaped stiffener as design variables, stress, the maximum deformation in vertical Y directions as the state variable, and select the weight of the reinforced wallboard as the objective function.

$$Min \;\; WT(X)$$

$$X = [X_1, X_2] = [H, T]$$

$$ABS(UY) = DEFL < 0.1^{-8}$$

$$STRESS \le \sigma_{0.2} / n = 410/4 = 102.5\text{MPa}$$

The range of design variables is:

$$0.1 \le S \le 5$$

$$0 < H < 0.05$$

$$0 < T < 0.02$$

The initial values of optimization are shown in Table 8.

Table 8. Preliminary values of optimization

Items	Values
Stiffener height (H)	0.007m
Stiffener width (T)	0.007m
Ratio of width to height (S)	1
weight(WT)	4.33867208kg
Maximum deformation(DEFL)	4.015569364E-09m

4.3 Optimization Results

The optimization method uses a zero-order method, which is an iterative approximation using all dependent variables (state variables and objective functions). The optimized iteration curve obtained from the 30 iterations is shown in Fig. 9. The results show that the 14th iteration is the optimal solution of the stiffened plate.

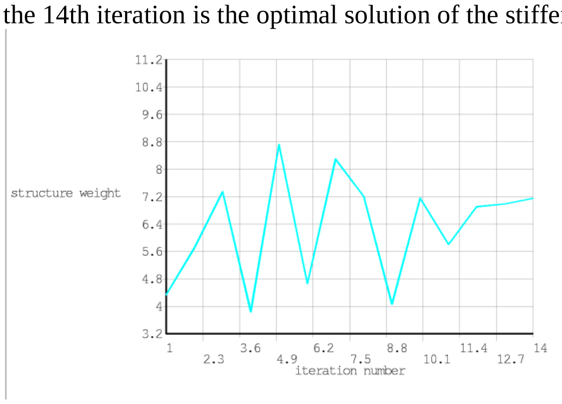


Fig. 9. Iterative Curve and Optimization Results

After the optimization calculation, the optimal solution is S=1.788, H=0.03m, and T=0.01m. And the maximum deformation is 0.58e-8m, the maximum stress is 1922.65Pa, as shown in Fig. 10 and Fig. 11.

5 Conclusions

In this article, four finite element models of stiffened thin plate structures with different cross-sectional shapes are established, and the effect of stiffener form and number on the intrinsic properties, vibration characteristics and yield load of thin plates are studied, and then take the stiffened wallboard with T-shaped stiffener as the research object, and its structural parameters of stiffeners are optimized and analyzed.

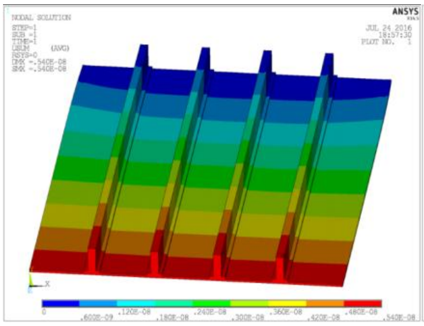


Fig. 10. Maximum deformation after optimization

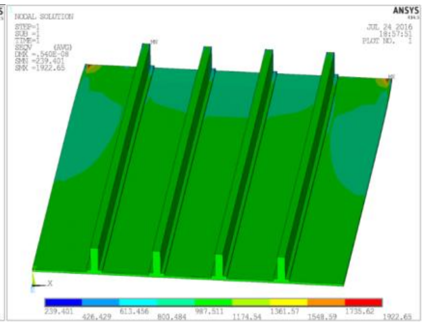


Fig. 11. Maximum stress after optimization

(1) The first 8-order natural frequencies of the stiffened plate with cap-shaped section are the highest, followed by the Z-shaped and the T-shaped stiffeners, and the first 8-order natural frequencies of the L-shaped stiffened plates are the lowest. With the number of stiffeners increasing, the natural frequencies of each order gradually increase.

(2) Vibration response of the first-order natural frequency of the reinforced plate with T-shaped stiffeners under the base excitation is the largest, The vibration reduction of the reinforced plate with Z-shaped stiffeners is better, followed by the cap-shaped stiffeners.

(3) Under pressure load conditions, the maximum stress of the reinforced plates exists at the two corners of the fixed edge. The maximum stress of thin plates from the largest to the smallest are ones with Z-shaped, T-shaped, cap-shaped, L-shaped stiffeners. The maximum stress of stiffened plates with different cross sections take occurs on the bars, when the cantilever ends are subjected to uniformly distributed line load. The maximum stress of thin plates from the largest to the smallest are ones with L-shaped, T-shaped, cap-shaped, Z-shaped ribs.

(4) Take the stiffened wallboard with T-shaped stiffener as the research object, and its structural parameters of stiffeners are optimized and analyzed: the optimized stiffened form can significantly reduce the maximum deformation and maximum stress of the thin plate structure. Therefore, in practical applications, it is necessary to find the most suitable stiffened structural parameters by optimization.

 ISSN 2572-4975 (Print), 2572-4991 (Online)

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