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Dynamic Similarity Design of Geared Rotor System in Five-Shaft Integrally Centrifugal Compressor

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Abstract: Geared rotor system is the core component of multi-shaft integrally centrifugal compressor, as well as the part with high failure rate. Considering the problems of high cost of experiment, and longtime cycle when direct using prototype machine, it is necessary to test its characteristic through a test rig with the same structure and same dynamics. This paper takes the rotor system of a five-shaft integrally centrifugal compressor as the research object. Through theoretical analysis and similar design theory, the design method of the five rotor systems are studied, and a similar model of the geared rotor system in integrally centrifugal compressor is obtained. At last, the validity of the scale model is confirmed by comparative analysis, which provides experimental basis for the future experimental research.

Keywords: Integrally centrifugal compressor; Geared rotor system; Dynamic similarity design; test rig

1 Introduction

Integrally centrifugal compressor is an advanced equipment widely used in energy, petrochemical and other fields. As the core component of integrally centrifugal compressor, its rotor system consists of a number of rotors meshing through gears, and long time working above critical speed with high load. Its dynamic characteristics are complex and cause high failure rate. The dynamic characteristics of the rotor system have direct influence on the dynamic characteristics of the whole system. At present, due to the limitations of the theoretical model, experimental analysis is still the best way to study the dynamic characteristics of the rotor system.

For a long time, the theoretical calculations are often used on the dynamic design of the gear rotor system but with few experimental data supported, which makes the designed system have a serious security risk and cause major economic losses [1]. Wehrman[2] pointed out that the dynamic problem of the rotor system is the key to

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the design of the whole compressor; Gruntfest[3] described that under high temperature operation the bearing of tilting tile solved the instability phenomenon of rotor system of multi-axes gear compressor and subsynchronous vibration of a rotor system through eddy current brake. Moore[4] proposed an idea that the stability of the rotor system would not be reduced with the increase of load. In order to better study the rotor dynamic characteristics of the five-axes gear assembly centrifugal compressor and solve the problem of difficult system testing, a corresponding experimental platform should be built using similar design theory. At present, Dynamic similarity theory has been widely applied in large scale structural dynamic test models. Harris[5] introduced the application of similarity principle and similar test model in structural analysis of bridges and buildings. Cho[6] proposed a distortion model test method for the distortion similar model test of composite structure system. Chinese scholar Hu Peimin [7] analyzed the similar model tests of rotor torsional vibration characteristics, and the results showed that for a rotor system with rigid ends at both ends, when its geometric size was scaled down to its original 1/n, its frequency characteristics and vibration modes were basically unchanged, and the natural frequency increased to its original n times. LUO Zhong[8,9] systematically studied the theory of similarity design, and proposed a dynamic similarity design method for multi-axes rotor bearing system distortion test model based on the transfer matrix method and sensitivity analysis, and the method was Verified.

At present, due to the larger size, compact structure, complex working environment and high rotating speed of integrally centrifugal compressor, using prototypes directly for testing costs too much time as well as money. Therefore, only theoretical analysis and numerical simulation are used in practical engineering design for theoretical guidance. It is necessary to establish a test rig with similar structural and similar dynamics.

In this paper, based on theoretical analysis and similar design theory, the design method of the scale test model for the integrally centrifugal compressor rotor system is studied, and the dynamic scale similarity model of geared rotor system is obtained, which can provide experiment basis for the dynamic characteristics research of integrally centrifugal compressor.

2 The modeling and similarity design method

2.1 The structural of the geared rotor system in integrally centrifugal compressor

The dynamic model of the gear system in integrally centrifugal compressor with five shafts is shown in Fig.1. The system consists of five parallel rotors meshing with helical gears and supported with bearings at each end. The input axis is connected to the steam turbine trough the coupling, and it can also be connected to the intermediate shaft using an additional gearbox.

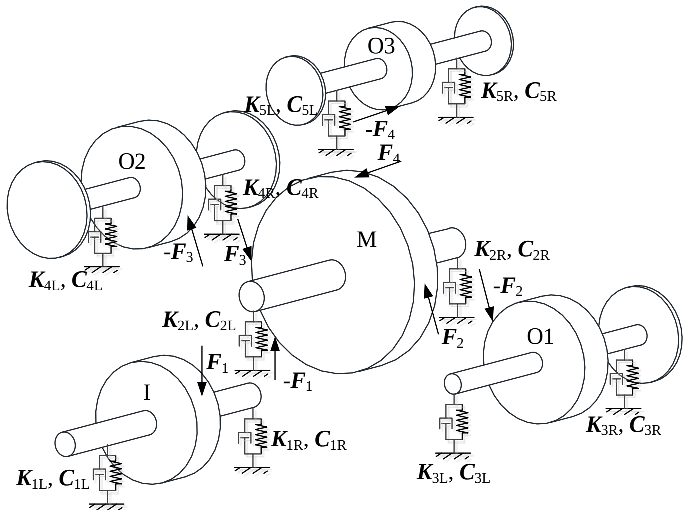

Fig.1 The dynamic model of the geared rotor system

The $\mathbf{K}_{iL}$, $\mathbf{C}_{iL}$ and the $\mathbf{K}_{iR}$, $\mathbf{C}_{iR}$ are used to stand for the stiffness and damping coefficient matrix of the left and the right supporting bearing of rotor *i*; the F_i is the meshing force of the gear.

2.2 The modeling method of the geared rotor system

For those rotor-bearing systems coupled by gear, the finite element method can be used to split the parts of the system into different finite elements for modeling and theoretical analysis.

There are several assumptions for the gear coupling rotor system linear modeling:

a. Bearing blocks and foundations are considered as rigid;

b. Assuming that the deformation of the shaft in the process of operation is very tiny, and the non-linear effects which caused by the axial deformation can be ignored, regarding the axis as a linear elastic axis;

c. Considering the impeller, wheel, driving gear and axial sections which have an larger size mutation as rigid disks;

d. Approximating the bearing to the linear spring damping element by neglecting the nonlinear characteristic of the bearing;

e. Approximate the gear meshing elements as linear spring damping elements by ignoring the nonlinear factors such as tooth profile error, tooth side gap of the gear.

In this model, each node has six degrees of freedom. The differential motion equation of the geared rotor system can be written as

$$\mathbf{M}\ddot{\mathbf{X}} + (\mathbf{C} + \Omega\mathbf{G})\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F} \tag{1}$$

where, $\mathbf{X}$ is the displacement vector of system node, $\mathbf{M}$ is the system mass matrix, $\mathbf{C}$ is the system damping matrix which includes the bearing damping and the internal

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damping of materials, **G** is the system gyroscopic moment matrix, Ω is the rotor speed, **K** is the system stiffness matrix, **F** is the force vector.

2.3 The similarity design method of the rotor system

The similarity design is a certain kind of special method which made the physical quantity of the design system is proportional to the physical quantity corresponding to the same phenomenon of the original system. According to the similarity principle of complex system, the similarity of the whole system will be guaranteed when the independent subsystems and the subsystems connecting them are similar at the same time. Therefore, for the geared rotor system in Fig.1, the similarity design of rotor system can be achieved by the single similar design for each rotor. For the rotor system, the differential vibration equation can be described as[11]:

$$\frac{\partial}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right)-\frac{\partial^2}{\partial x^2}\left(a\frac{\partial^2 y}{\partial t^2}\right)+2i\omega\frac{\partial^2}{\partial x^2}\left(a\frac{\partial y}{\partial t}\right)+m\frac{\partial^2 y}{\partial t^2}=p(x)e^{i\omega t} \quad (2)$$

where, $m(x)$ is the the mass of the unit length of the rotor, E is the elastic modulus of rotating shaft, I is the section moment of inertia of the shaft, $a(x)$ is the moment of inertia of unit length relative to rotation axis (the moment of inertia of the axis can be ignored), y is the synthetic deflection of the axis, $p(x)$ is the strength of the rotor unbalance force, $p(x)=me(x)\omega^2$, $e(x)$ is the imbalance curve, ω is the rotational frequency of the rotor.

As supplement to the Eq. (2), the expression of the inclination angle and normal stress of the axis elastic line can be written as

$$\begin{aligned}\alpha &= \mathrm{d}y/\mathrm{d}x \\ \sigma &= M_*/W\end{aligned} \quad (3)$$

where, M_* is the bending moment, W is the coefficient of flexural section.

When designing the model, selecting those values which can actually be changed independently of each other as independent values, then the scale of the remaining values can be found in the similarity index corresponding to the criteria.

On the premise of meeting the requirements of rotor dynamics, it is also necessary to simplify the structure of the complex rotor.

The the critical speed increases with the increase of the flexural stiffness of the shaft section, the sensitivity of the *ith* critical speed to the bending stiffness EI of the *jth* shaft section can be written as

$$\frac{\partial N_i}{\partial (EI)_i}=\frac{1800}{\pi^2 N_i l_j^3}\left[3\left(\bar{y}_j-\bar{y}_{j+1}\right)^2+3l_j\left(\bar{y}_j-\bar{y}_{j+1}\right)\left(\bar{\theta}_j-\bar{\theta}_{j+1}\right)+l_j^{\,2}\left(\bar{y}_j-\bar{y}_{j+1}\right)\left(\bar{\theta}_j^{\,2}+\bar{\theta}_j\bar{\theta}_{j+1}+\bar{\theta}_{j+1}^{\,2}\right)\right]$$

$$(i=1,2,3,\cdots;\ j=1,2,\cdots,N-1) \quad (4)$$

In this formula, l_j is the length of the *jth* axis, N is the number of nodes, the $\bar{y}_j$, $\bar{y}_{j+1}$ and the $\bar{\theta}_j$, $\bar{\theta}_{j+1}$ respectively means the regularized linear displacement and the angular displacement amplitude value of the left and right nodes of the *jth* shaft at the mainly *i*th vibration mode.

The influence of shaft length on critical speed is exist in both positive and negative side. The sensitivity of the *ith* critical speed to the length of the *jth* axis can be expressed as

$$\frac{\partial N_i}{\partial l_i} = -\frac{1800}{\pi^2 N_i}\left(\frac{EI}{l^4}\right)_i \left[9\left(\bar{y}_j - \bar{y}_{j+1}\right)^2 + 6l_j\left(\bar{y}_j - \bar{y}_{j+1}\right)\left(\bar{\theta}_j + \bar{\theta}_{j+1}\right) + l_j^{\,2}\left(\bar{\theta}_j^{\,2} + \bar{\theta}_j\bar{\theta}_{j+1} + \bar{\theta}_{j+1}^{\,2}\right)\right]$$

$$(i = 1, 2, 3, \cdots;\ j = 1, 2, \cdots, N-1) \qquad (5)$$

The critical speed decreases with the increase of the mass of nodes, the susceptibility of the the *ith* critical speed to the mass of the *jth* node and the moment of inertia $J_p - J_d$ can be expressed as

$$\begin{aligned} \frac{\partial N_i}{\partial m_i} &= -\frac{N_i}{2}\bar{y}_j^{\,2} \\ \frac{\partial N_i}{\partial\left(J_p - J_d\right)_i} &= -\frac{N_i}{2}\bar{\theta}_j^{\,2} \end{aligned} \qquad (i = 1, 2, 3, \cdots;\ j = 1, 2, \cdots, N-1) \qquad (6)$$

If the *qth* critical speed needs to be changed, the change amount is

$$\Delta \boldsymbol{N} = \left[\Delta N_1 \quad \Delta N_2 \quad \cdots \quad \Delta N_q\right]^T \qquad (7)$$

The existing n parameters p_i ($i = 1, 2, \cdots, \mathrm{n}$)is allowed to have an appropriate adjustments, assume the change of the parameters is :

$$\Delta \boldsymbol{P} = \left[\Delta P_1 \quad \Delta P_2 \quad \cdots \quad \Delta P_n\right]^T \qquad (8)$$

So there is

$$\boldsymbol{J}\Delta\boldsymbol{P} = \Delta\boldsymbol{N} \qquad (9)$$

where, $\boldsymbol{J}$ is the Jacobi matrix of the order $q \times n$, concretely can be written as formula (10), and the matrix elements are the sensitivity of the critical speed to each parameter.

$$\boldsymbol{J} = \frac{\partial\left(N_1, N_2, \cdots, N_q\right)}{\partial\left(p_1, p_2, \cdots, p_n\right)} = \begin{bmatrix} \frac{\partial N_1}{\partial p_1} & \frac{\partial N_1}{\partial p_2} & \cdots & \frac{\partial N_1}{\partial p_n} \\ \frac{\partial N_2}{\partial p_1} & \frac{\partial N_2}{\partial p_2} & \cdots & \frac{\partial N_2}{\partial p_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial N_q}{\partial p_1} & \frac{\partial N_q}{\partial p_2} & \cdots & \frac{\partial N_q}{\partial p_n} \end{bmatrix} \qquad (10)$$

According to the Eq.(9) and Eq.(10), if $q < n$, the Eq.(9) could have infinitely many solutions. Meanwhile, the sensitivity calculation formulas of the Eqs. (2), (5) and (6) are obtained by ignore the trace above the second order, thus multiple optimization to reduce error is necessary.

3 Design and verification of the integrally centrifugal compressor test rig

3.1 Similar design of the integrally centrifugal compressor test rig

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The similar design of the rotor system is achieved by individually similar designs of each rotor. The design data of each rotor of the prototype are shown in Table 1, where I, M, O1, O2 and O3 stand for Input shaft, Intermediate shaft, Output shaft 1, Output shaft 2 and Output shaft 3 respectively.

Table 1. Main parameters of five-axis gear assembly compressor

	pitch diameter	Trans-mission ratio	Working Revolution		1st critical speed		2nd critical speed	
	D(mm)	*i*	n(r/min)	Hz	(r/min)	Hz	(r/min)	Hz
I	719	1.000	4087	68.12	10513	175.2	--	--
M	1984	0.362	1481	24.68	5865	97.8	--	--
O1	264	2.723	11131	185.52	8471	141.2	19626	327.1
O2	221	3.253	13297	221.62	9287	154.8	10083	168.1
O3	141	5.099	20841	347.35	11446	190.8	12919	215.3

The similar model of the rotor system should satisfy similar principles.

a. **Geometrical similarity**. The spatial arrangement angle of each parallel shaft rotor is similar to that of the prototype.The pressure angle and tilt angle of gear are consistent with the prototype;

b. **Kinematic similarity**. The transmition gear ratio of each gear is similar to that of the prototype.

c. **Dynamical similarity**. The working speeds of the input shaft and the intermediate shaft are below the 1st critical speed, the operating speed of the output shaft 1 is between its 1st critical and 2nd critical speed, and the operating speeds of the output shaft 2 and the output shaft 3 are both between the 2nd critical and the 3rd critical.

According to the dynamic similarity design method of distortion model of multi-segment rotor system, the materials which are consistent with that of the prototype rotor to do similarity design are used.Then do dynamic modeling to obtain the distortion model. At last, according to the Eq.(10), dynamic correction is done to make the critical speed similarity ratio approximately satisfy the equation $\lambda_\omega = 2$. The result is shown in Figure 2.

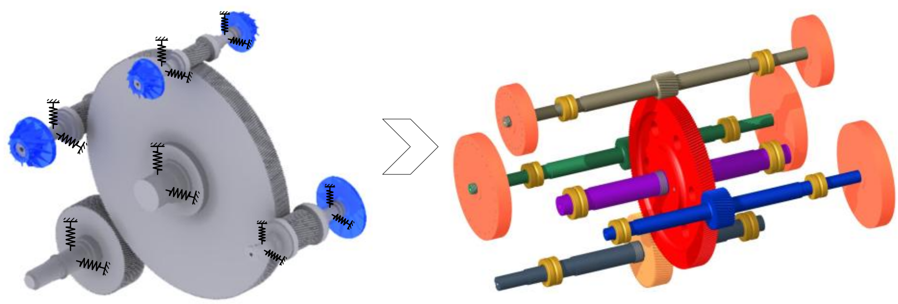


Fig. 2. Similar design results for a five-axis compressor rotor system

3.2 Numerical verification of the dynamic characteristics of the test rig

By the method of uniaxial similarity verification, the vibration modes and Campbell diagrams of the input shaft, intermediate shaft, and three output shafts of the prototype and scale model are calculated. The results are shown in Tables 2-6.

Table 2. Comparison between intrinsic characteristics of the prototype input shaft and the scale model input shaft

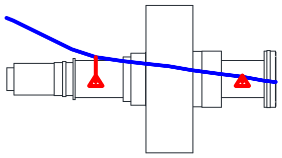
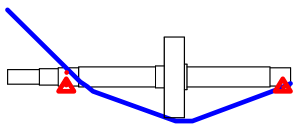
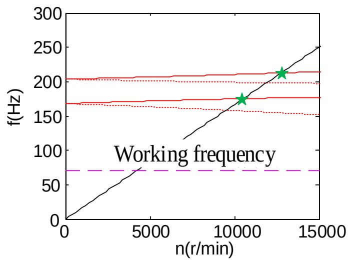
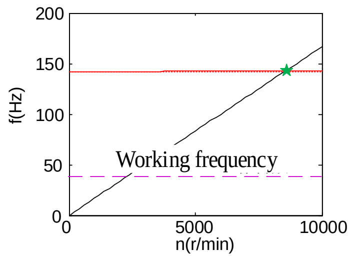
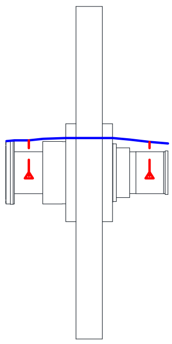
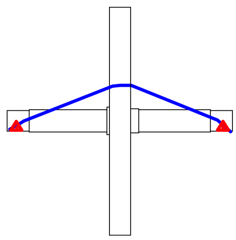
	prototype	Scale model
The 1st vibration mode		
campbell diagram		

Table 3. Comparison of inherent characteristics of the prototype intermediate shaft and scale model intermediate shaft

	prototype	Scale model
The 1st vibration mode		

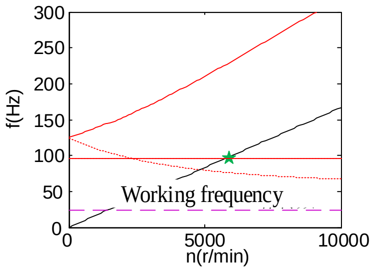
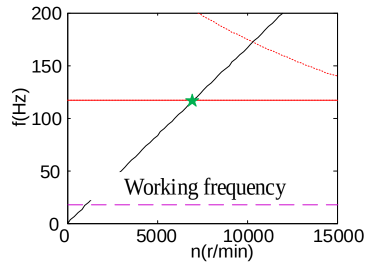
Campbell diagram		

Table 4. Comparison of the inherent characteristics of the prototype output shaft 1 and scale model output shaft 1

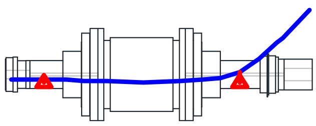
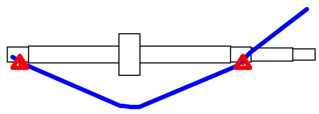
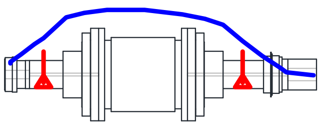
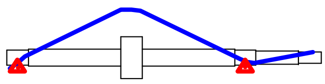
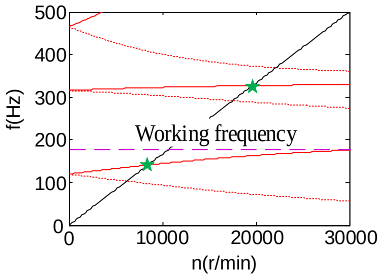
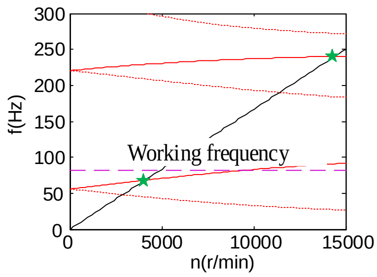
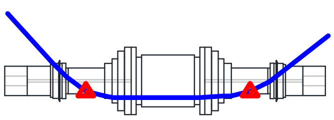
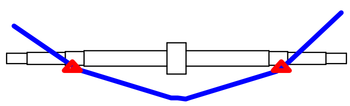
	prototype	Scale model
The 1st vibration mode		
The 2nd vibration mode		
Campbell diagram		

Table 5. Comparison of inherent characteristics of prototype output shaft 2 and scale model output shaft 2

	prototype	Scale model
The 1st vibration mode		

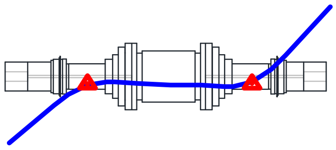
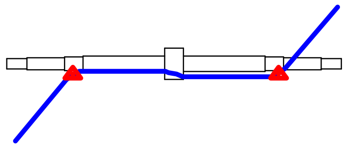
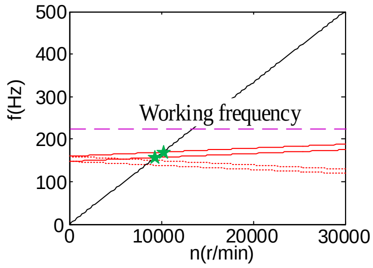
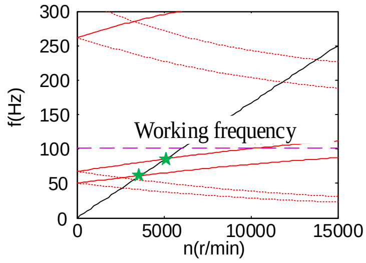
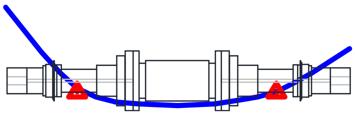
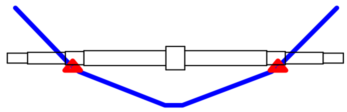
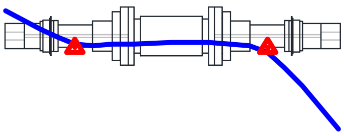
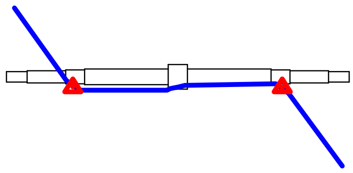
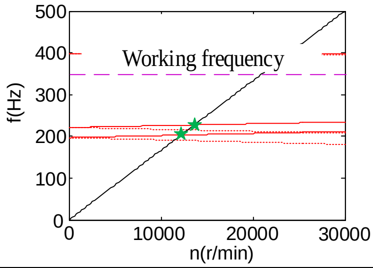
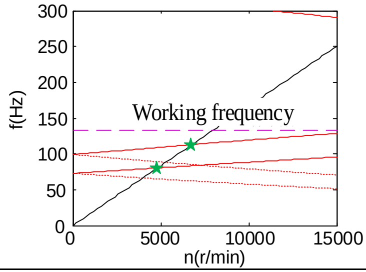
The 2nd vibration mode		
Campbell diagram		

Table 6. Comparison of Intrinsic Characteristics of Prototype Output Shaft 3 and Scale Model Output Shaft 3

	prototype	Scale model
The 1st vibration mode		
The 2nd vibration mode		
Campbell diagram		

According to Table 2~6, the vibration mode and the Campbell diagram of the prototype rotor is basically consistent with that of its similar rotor below the operating speed, and there is a good similar relationship betweeen the two rotors. According to the similarity principle of the complex system, the test-rig rotor system obtained

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through similar design can reflect the dynamic characteristics of the prototype rotor system.

The main parameters of each rotor in the obtained rotor system are shown in Table 7.

Table 7. Main parameters of the test rig

	Number of teeth	Gear Diameter	Transmission ratio	Work Revolution		1st critical speed		2nd critical speed	
	z	D(mm)	i	n(r/min)	Hz	(r/min)	Hz	(r/min)	Hz
I	80	205.13	--	2500	41.7	8297	138.3	--	--
M	192	494.87	0.417	1042	17.4	7089	118.2	--	--
O1	41	105.13	1.951	4878	81.3	4004	66.7	14421	240.4
O2	33	84.69	2.422	6061	101	3592	59.9	5108	85.1
O3	25	64.22	3.194	8000	133.3	4794	79.9	6826	113.8

From Table 7, it can be seen that each of the output shafts satisfies the requirement of dynamic similarity when each rotor rotates up to the working speed.

4 Conclusions

This paper is proposed a similarity design method for the geared rotor system in integrally centrifugal compressor based on similarity design principles. According to similarity design method of the geometric distortion, the dynamic characteristics of the scaled test rig below the working speed are same with the prototype. By comparing the vibration model and Campbell diagram between the prototype rotor system and test rig rotor system, it shows that the rotor system obtained by the similarity design method has similar dynamic characteristics at the working speed, which meets the basic requirements of the integrally centrifugal compressor's dynamics experiments.

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