Stiffness Characteristics of a Special-shaped Rolling Bearing

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Abstract: In order to study the structural and mechanical characteristics of special-shaped bearings for aeroengine composite elastic support structures, a five DOF analytical model of special-shaped angular contact ball bearings was established by quasi-static analysis method based on Hertz contact theory and stiffness ferrule hypothesis. The finite element models of common angular contact ball bearings and special angular contact ball bearings are established by ANSYS, and the stiffness characteristics of common angular contact ball bearings and special angular contact ball bearings are analyzed and compared by finite element method considering the influence of bearing axial preload.

Keywords: Special-shaped bearing; Stiffness characteristics; Analytical model; Finite element model

1 Introduction

As the core force-bearing and force-transmitting components of aero-engine rotor system, elastic support structure usually consists of special-shaped rolling bearings with mounting edges on the outer ring and squirrel-cage elastic rings. Not only to meet the structural requirements of simple structure, light weight, convenient processing and assembly, but also to meet the mechanical requirements of good damping characteristics and vibration isolation. The stiffness characteristics of elastic

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support structures, especially special-shaped bearings, have a great influence on the dynamic performance of the rotor system. At the same time, rotor vibration also has an extremely important influence on the stiffness of special-shaped bearings and the behavior of their elastic support structures.

For the study of rolling bearing stiffness, Noel [1] proposed a stiffness matrix calculation method for 5-DOF angular contact ball bearings based on Jones model. Tang Yunbing[2,3] established the mechanical model of rolling bearing by quasi-static method and finite element method respectively to study the influence of rolling bearing structure and load on antenna, deformation and contact stiffness. Wang Shuogui et al [4] based on quasi-static analysis and raceway control theory, analyzed the influence of installation interference and pre-tightening force on radial, axial and angular stiffness of high-speed angular contact ball bearings. Yi [5] studied the influence of different axial preload and rotational speed conditions on the contact parameters and stiffness of angular contact ball bearings based on quasi-static analysis method, and verified the accuracy of the model by testing the displacement of inner ring and outer ring.

At present, there is little research on the stiffness characteristics of special-shaped bearings. The mechanical properties of the bearings with two different flange edges are compared and analyzed [6] and the bearing structural design optimization is guided by the stress distribution characteristics of the bearings. Wang Jintang [7] and others calculated the axial bearing capacity of the flange outer ring special-shaped four-point contact ball bearing through theoretical and numerical simulation. In the existing research results, the theoretical and experimental research on bearing stiffness characteristics considering the structural characteristics, bolt connection state and working conditions of the aerogenerator special bearing with flange mounting edge is insufficient.

In this paper, the stiffness analysis model and finite element analysis software are established for a typical aeroengine elastic support structure, and the stiffness characteristics of common angular contact ball bearings and special-shaped angular contact ball bearings are compared and analyzed.

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2 Mechanical model of bearing stiffness

2.1 Force Analysis of Roller

Ignoring the influence of bearing ball centrifugal force, internal friction and friction torque, the overall mechanical characteristics of the bearing are analyzed based on the assumption of rigid ring, assuming the number of special-shaped bearing rolling elements m, each rolling element interacts with the inner and outer raceway through point contact, and its mechanical model principle is shown in Fig. 1. The contact state of j-th rolling element of the rolling bearing with the inner and outer rings is schematically shown in Fig. 2. The coordinate system is set to OXr , OX and Or represent the axial direction and the radial direction. The initial contact angle between the rolling element and the inner and outer rings is α_0 O_i represents the center of curvature of the inner ring groove before loading and O_o is the center of curvature of the outer ring groove before loading.

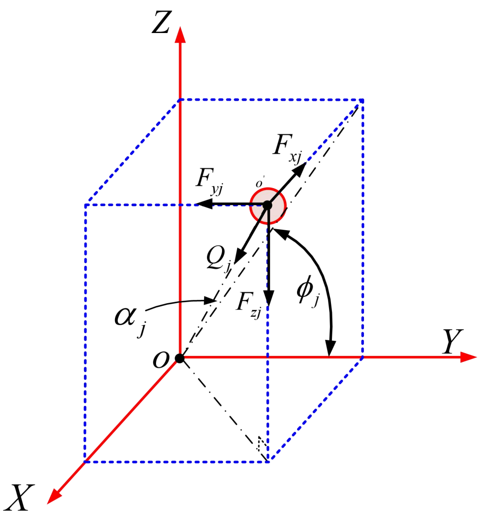

Fig.1. Mechanical model of one ball

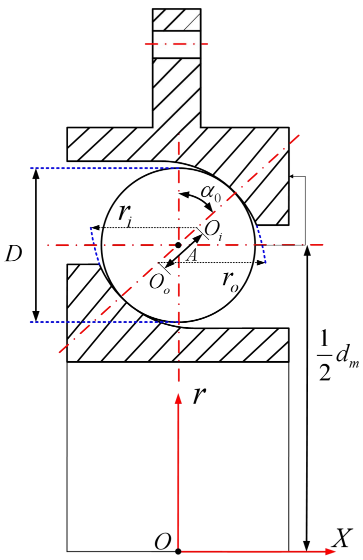

Fig.2. Inner and outer raceway and j th ball contact

Assuming that the $A0$ is initial distance between O_i the center of curvature of the inner and O_o the center of curvature of the outer race grooves of the bearing before loading, as can be seen from Fig. 2, it can be expressed as

$$A_o = r_o + r_i - D = (f_o + f_i - 1)D \tag{1}$$

Where, D is Roller diameter, r_i and r_0 the groove curvature radius of the inner and outer rings of the bearing, f_i and f_0 are the groove curvature radius coefficients of the inner ring and the outer ring of the bearing respectively. For ball bearings, the value range of f_i and f_0 are set between 0.515 and 0.525.

As shown in Fig. 3, the deformation of bearing roller and inner/outer rings during initial installation and pre-tightening is schematically shown. The contact angle between the roller and the inner and outer rings changed from the initial value α_0 to α_i and the center of curvature of the inner ring groove changed from the initial position O_i to $O_i^{'}$. Assuming the bearing flange outer ring is fixed, the center position of the curvature of the bearing outer ring groove before and after loading remains unchanged, still in O_o. Set the distance between $O_i^{'}$ center position of the curvature of the groove before inner ring is loaded and O_o center position of the curvature of the groove after inner ring is loaded is A_j, the total amount of contact deformation of the j-th rolling element in the contact normal direction is δ_j, and δ_j the total amount of contact deformation consists of two components: δ_{xj} axial elastic deformation and δ_{rj} radial elastic deformation.

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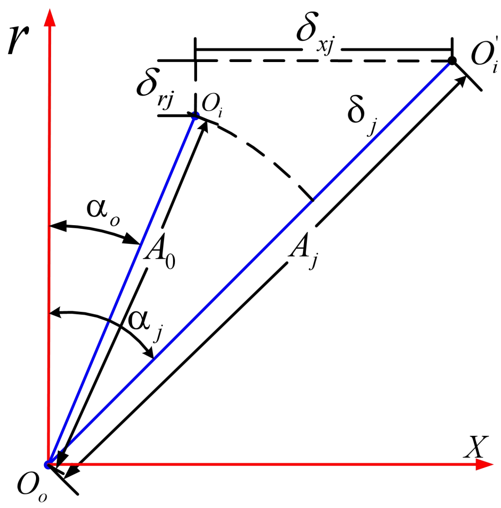

Fig.3. Relative deformation of inner and outer raceway and ball

As can be seen from Fig. 3, the distance between O_i' center position of the curvature of the groove before inner ring is loaded and O_o center position of the curvature of the groove after inner ring is loaded are expressed as:

$$A_j = \sqrt{(A_0 \sin(\alpha_0) + \delta_{xj})^2 + (A_0 \cos(\alpha_0) + \delta_{rj})^2} \tag{2}$$

When the angular contact bearing is not subjected to centrifugal load, the initial contact angles of the inner and outer raceways are the same. The size of the initial contact angle is related to the total curvature, roller diameter and installation clearance under no load. The expression is:

$$\alpha_0 = \arccos(1 - \frac{P_d}{2(f_o + f_i - 1)D}) \tag{3}$$

Where, P_d is the installation clearance, the contact angle of the bearing changes when it is subjected to a load. As can be seen from Fig. 3, the actual contact angle between j th roller and the bearing inner/outer rings after loading becomes:

$$\alpha_j = \arctan\frac{A_0\sin(\alpha_0)+\delta_{xj}}{A_0\cos(\alpha_0)+\delta_{rj}} \tag{4}$$

The relative deformation diagram of the j th rolling element and the inner and outer rings of the bearing based on the analytical model of the 5 DOF bearings is shown in Fig. 4.

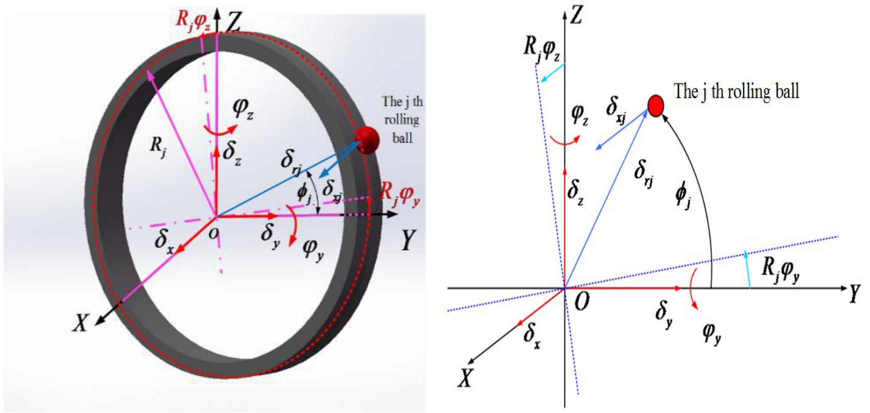

Fig.4. Inner and outer raceway and j th ball deformation

As shown in Fig. 4, the azimuth of the j th rolling element is ϕ_j , and the expression of total contact deformation is δ_j , can be expressed:

$$\delta_j = \begin{Bmatrix} A_j - A_0, \delta_j > 0 \\ 0, \qquad \delta_j \le 0 \end{Bmatrix} \tag{5}$$

The expressions of δ_{xj} axial elastic deformation and δ_{rj} radial elastic deformation at the j th rolling element are respectively:

$$\delta_{xj} = \delta_x + R_j(\varphi_y \sin\phi_j - \varphi_z \cos\phi_j) \tag{6}$$

$$\delta_{rj} = \delta_y \cos\phi_j + \delta_z \sin\phi_j \tag{7}$$

Where, R_j represents the center track radius of raceway groove curvature , and its

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expression is:

$$R_j = \frac{d_m}{2} + (r_i - \frac{D}{2})\cos\alpha_0 \tag{8}$$

2.2 Load - displacement relationship of bearings

The force balance equation of the j-th rolling element at the azimuth angle is ϕ_j [8]:

$$\begin{cases} F_{xj} = Q_j \sin\alpha_j \\ F_{yj} = Q_j \cos\alpha_j \cos\phi_j \\ F_{zj} = Q_j \cos\alpha_j \sin\phi_j \\ M_{yj} = R_j Q_j \sin\alpha_j \sin\phi_j \\ M_{zj} = -R_j Q_j \sin\alpha_j \cos\phi_j \end{cases} \tag{9}$$

According to the force balance condition of the bearing inner ring, the external force acts on the bearing inner ring through the rotating shaft. According to the principle of mechanical balance, the external force counteracts the force of all rolling elements on the bearing inner ring. The force balance equation of the bearing as a whole can be obtained by superimposing and summing all the forces between the rolling elements and the bearing inner ring.

$$\begin{cases} F_x = \sum_{j=1}^{m} Q_j \sin\alpha_j \\ F_y = \sum_{j=1}^{m} Q_j \cos\alpha_j \cos\phi_j \\ F_z = \sum_{j=1}^{m} Q_j \cos\alpha_j \sin\phi_j \\ M_y = \sum_{j=1}^{m} R_j Q_j \sin\alpha_j \sin\phi_j \\ M_z = -\sum_{j=1}^{m} R_j Q_j \sin\alpha_j \cos\phi_j \end{cases} \tag{10}$$

Where, $\{F_x \quad F_y \quad F_z \quad M_y \quad M_z\}^T$ indicates the external force acting on the bearing inner ring through the rotating shaft, Q_j is the normal contact load between the j th rolling element and the inner raceway of the bearing, α_j the contact angle between the j th rolling element and the inner and outer rings under the external load is calculated as shown in Equation (2) and the calculation as shown in Equation (7).

2.3 Derivation of Stiffness Matrix of Special Bearing

Considering the special-shaped bearing at the mounting edge of the outer ring, the outer ring of the bearing is fixed by bolt connection, and the matching relationship between the inner ring and the rotating shaft is selected as interference, resulting in a simplified mechanical model of the special-shaped bearing as shown in Fig. 5.

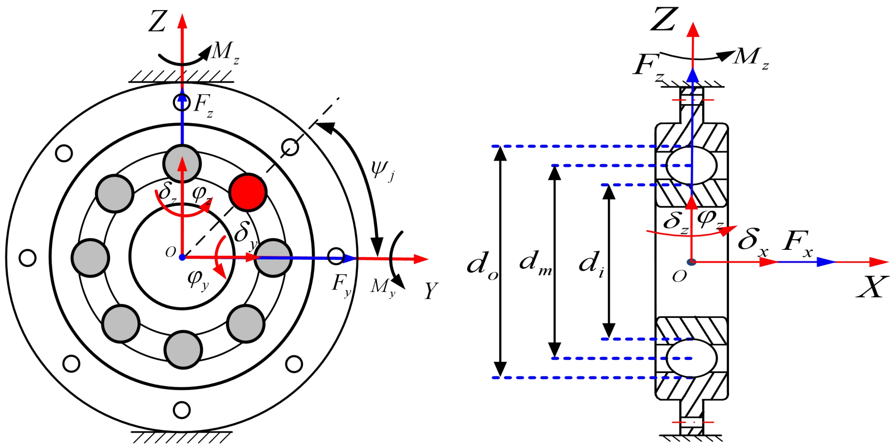

Fig.5. Simplified mechanical model of rolling bearing

If the bearing is considered as a stiffness matrix model, the load-displacement relationship of the bearing can be expressed as

$$\boldsymbol{F} = \boldsymbol{K}\boldsymbol{q} \tag{11}$$

Where, $\boldsymbol{F} = \{F_X \quad F_Y \quad F_Z \quad M_Y \quad M_Z\}^{\mathrm{T}}$ indicates the external force acting on the inner ring of the special-shaped bearing through the rotating shaft,

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$q = \{\delta_X \quad \delta_Y \quad \delta_Z \quad \varphi_Y \quad \varphi_Z\}^T$ represents the displacement of the bearing under the corresponding load, and derives the partial derivative of F the external force on the q bearing displacement， ignoring the cross stiffness, and gets 5 × 5 bearing stiffness matrix K [9,10],, its expression is:

$$\boldsymbol{K} = \begin{bmatrix} \frac{\partial F_x}{\partial \delta_x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial F_y}{\partial \delta_y} & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial F_z}{\partial \delta_z} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial M_y}{\partial \varphi_y} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial M_z}{\partial \varphi_z} \end{bmatrix} = \begin{bmatrix} K_{11} & 0 & 0 & 0 & 0 \\ 0 & K_{22} & 0 & 0 & 0 \\ 0 & 0 & K_{33} & 0 & 0 \\ 0 & 0 & 0 & K_{44} & 0 \\ 0 & 0 & 0 & 0 & K_{55} \end{bmatrix} \tag{12}$$

The elements on the diagonal represent the common bearing stiffness index - bearing main stiffness. Based on the established bearing load-displacement relation (12), the calculation formula of each main stiffness in the K bearing stiffness matrix is derived, K_{11} representing the bearing axial stiffness matrix, K_{22} representing the bearing Y radial stiffness matrix, K_{33} representing the bearing Z radial stiffness matrix, K_{44} representing the bearing Y directional stiffness matrix and K_{55} representing the bearing Z directional stiffness matrix. The specific expressions of each main stiffness are:

$$\begin{cases} K_{11} = K_n \sum_{j=1}^{m} \dfrac{(A_j - A_0)^n \left[\dfrac{nA_j(A_0 \sin\alpha_0 + \delta_{xj})^2}{A_j - A_0} + A_j^2 - (A_0 \sin\alpha_0 + \delta_{xj})^2 \right]}{A_j^3} \\ K_{22} = K_n \sum_{j=1}^{m} \dfrac{(A_j - A_0)^n \cos^2\phi_j \left[\dfrac{nA_j(A_0 \cos\alpha_0 + \delta_{rj})^2}{A_j - A_0} + A_j^2 - (A_0 \cos\alpha_0 + \delta_{rj})^2 \right]}{A_j^3} \\ K_{33} = K_n \sum_{j=1}^{m} \dfrac{(A_j - A_0)^n \sin^2\phi_j \left[\dfrac{nA_j(A_0 \cos\alpha_0 + \delta_{rj})^2}{A_j - A_0} + A_j^2 - (A_0 \cos\alpha_0 + \delta_{rj})^2 \right]}{A_j^3} \\ K_{44} = K_n \sum_{j=1}^{m} \dfrac{R_j^2(A_j - A_0)^n \sin^2\phi_j \left[\dfrac{nA_j(A_0 \sin\alpha_0 + \delta_{xj})^2}{A_j - A_0} + A_j^2 - (A_0 \sin\alpha_0 + \delta_{xj})^2 \right]}{A_j^3} \\ K_{55} = K_n \sum_{j=1}^{m} \dfrac{R_j^2(A_j - A_0)^n \cos^2\phi_j \left[\dfrac{nA_j(A_0 \sin\alpha_0 + \delta_{xj})^2}{A_j - A_0} + A_j^2 - (A_0 \sin\alpha_0 + \delta_{xj})^2 \right]}{A_j^3} \end{cases} \tag{13}$$

In practical application, if q the bearing displacement is known, the main stiffness of the bearing can be directly calculated by the above formula (13); If F the external force on the bearing is known, q the displacement of the bearing can be derived from the nonlinear equations (10) and the main stiffness of the bearing can be calculated by the above equation (13).

3 Finite Element Model of Stiffness of Heterosexual Angular Contact Ball Bearings

Taking the special 7013 AC angular contact ball bearing with flange mounting edge as the analysis object, without considering rolling friction, centrifugal effect and lubrication, the finite element model of the bearing is statically analyzed by ANSYS software to obtain the contact stress and deformation of the bearing, as shown in Fig.

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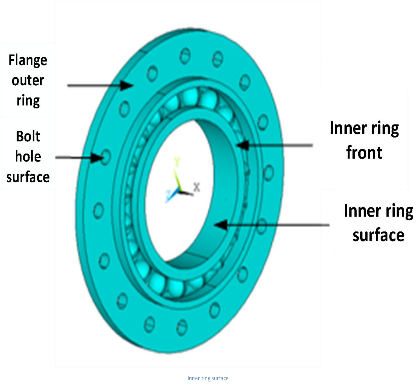
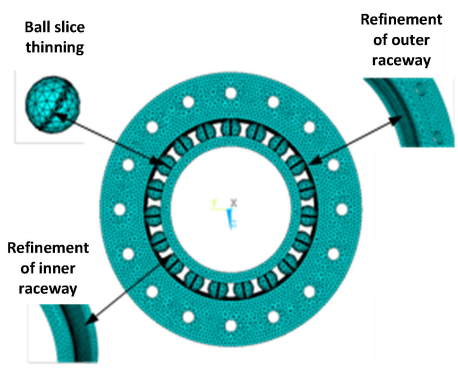
6 (a). In the process of modeling, the same simplified method as the ordinary angular contact ball bearing in the upper section is adopted, ignoring the transition fillet and chamfer.

The bearing inner and outer rings and rolling elements are made of GCR 15 bearing steel with the material parameters of 7.8g/cm3 density, Poisson's ratio of 0.29 and elastic modulus of 207 GPA. Solid 186 solid unit is used for free grid division, the grid unit size of the non-contact area inner and outer rings, left and right sides of the ball is 2mm, the contact area grid is refined to ensure contact accuracy, the inner and outer raceways are refined by 0.4 mm, the ball middle contact area is 0.3 mm, there are 592530 nodes, and there are 375951 units. The finite element model of the special 7013 AC angular contact ball bearing is shown in Fig. 6 (b), the contact setting is the same as the upper joint, and the contact pairs are set between the inner and outer rings of the bearing and the ball contact area. The constraint setting is shown in Fig. 7 in order to limit the displacement and rotation of the bearing flange outer ring, the bolt hole is fully constrained. In order to limit the position of the rolling element, the rolling element ball is circumferentially constrained; Apply an axial force to each node on the front of the bearing inner ring, and the sum of the axial forces of the nodes is defined as the axial load. Half of the inner ring surface of the bearing inner ring is coupled to the center node and a radial load is applied to the center node.





(a) Finite element model (b) Grid division

Fig.6. Finite element model diagram of special 7013AC angle contact ball bearing

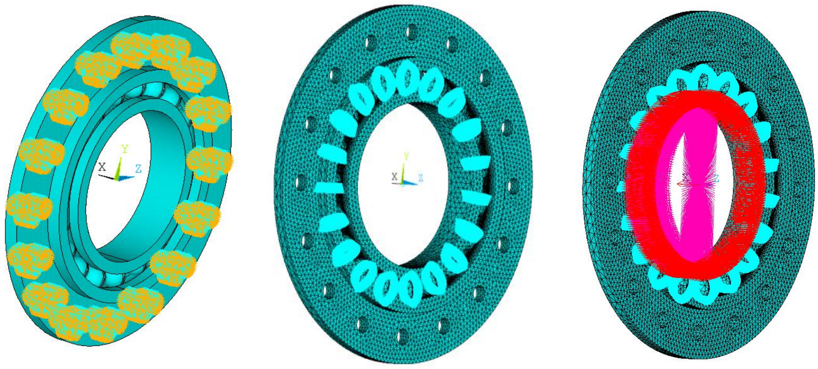


Fig.7. Constraints and load settings for special 7013AC angle contact ball bearing

4 Analysis and Comparison of Influence of Bearing Stiffness

4.1 Analysis of Influence of Outer Ring Flange Edge on Bearing Stiffness

According to the above-mentioned analytical model of bearing stiffness and the established finite element model of bearing stiffness, taking 7013 AC angular contact ball bearings as an example, the stiffness characteristics of ordinary angular contact ball bearings and special-shaped angular contact ball bearings with flange mounting edges are compared and analyzed without considering the influence of radial force and centrifugal force. The influence of the outer ring flange on the deformation and overall stiffness of the special angular contact ball bearing is emphatically analyzed.

The specific parameters of a common 7013 AC angular contact ball bearing are shown in Table 1.

(1) Influence of radial stiffness

Given a smaller axial preload, the radial load of the bearing is constantly changed, the radial stiffness and radial displacement of the bearing are obtained analytically, and the radial displacement of the bearing is obtained by finite element method. Through the load-displacement-stiffness relationship, the radial displacement and axial stiffness of the bearing under radial load are fitted as shown in Fig. 8-9.

According to the radial displacement and stiffness of the bearing under different radial loads shown above, it can be seen that:

1) With the increase of radial load, the radial displacement of the bearing increases and the radial stiffness of the bearing decreases.

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Table.1. Parameters of 7013AC angular contact ball bearing

Parameter	Value
Diameter inner ring raceway d_i	71.4mm
Curvature radius of inner ring groove r_i	5.7165mm
Curvature radius of outer ring groove r_o	5.7165mm
Roller diameter D	11.1mm
Inner diameter of bearing inner ring D_i	65mm
Bearing outer ring diameter D_o	100mm
Contact angle α^o	25
Number of rolling elements Z	19
Bearing width B	18mm

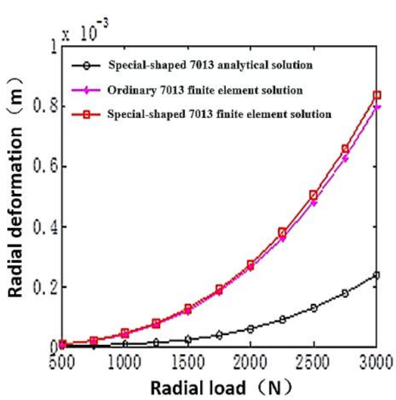


Fig.8. Radial deformation under different radial forces

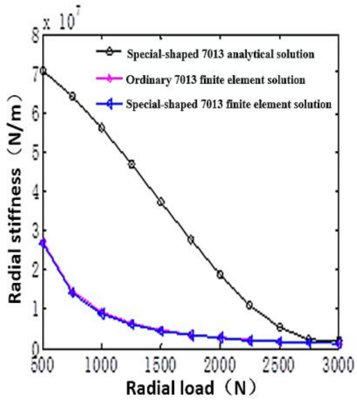


Fig.9. Radial stiffness under different radial forces

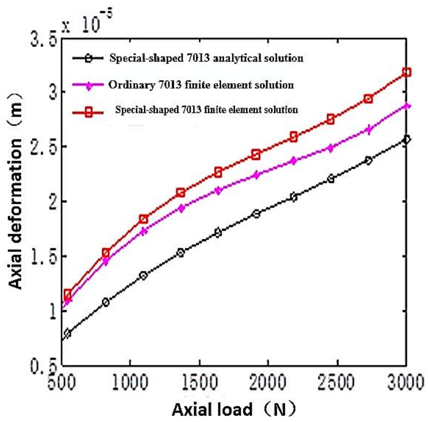
2) The radial displacement finite element result of the ordinary 7013 AC angular contact ball bearing is larger than the analytical result and slightly smaller than the radial displacement finite element result of the special 7013 AC angular contact ball bearing.

3) The radial stiffness finite element results of ordinary 7013 AC angular contact ball bearings are smaller than the analytical results and slightly larger than the radial stiffness finite element results of special 7013 AC angular contact ball bearings.

4) The flange edge of the outer ring of the special-shaped bearing has little contribution to the radial displacement and radial stiffness of the bearing, and only plays the role of structural connection.

(2) Influence of axial stiffness

Given a small radial load, the axial pre-tightening of the bearing is constantly changed, the axial stiffness and axial displacement of the bearing are obtained analytically, and the axial displacement of the bearing is obtained by finite element method. the axial displacement and axial stiffness of the bearing under axial load as shown in Fig.10-11 are fitted through the load - displacement - stiffness relationship.



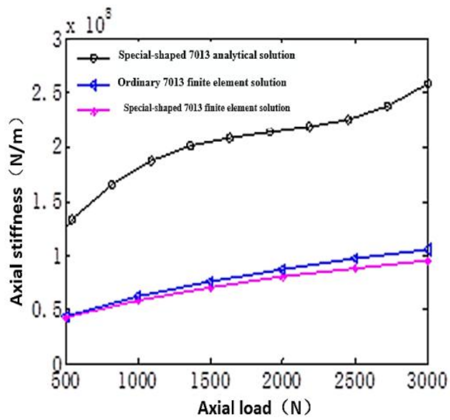


Fig.10. Axial deformation under axial force **Fig.11.** Axial stiffness under axial force

According to the axial displacement and stiffness of the bearing under different axial loads shown above, it can be seen that:

1) With the increase of axial load, the axial displacement of the bearing increases and the axial stiffness of the bearing increases.

2) The axial displacement finite element result of the ordinary 7013 AC angular contact ball bearing is larger than the analytical result and slightly smaller than the axial displacement finite element result of the special 7013 AC angular contact ball bearing.

3) The axial stiffness finite element results of ordinary 7013 AC angular contact ball bearings are smaller than the analytical results and slightly larger than the axial

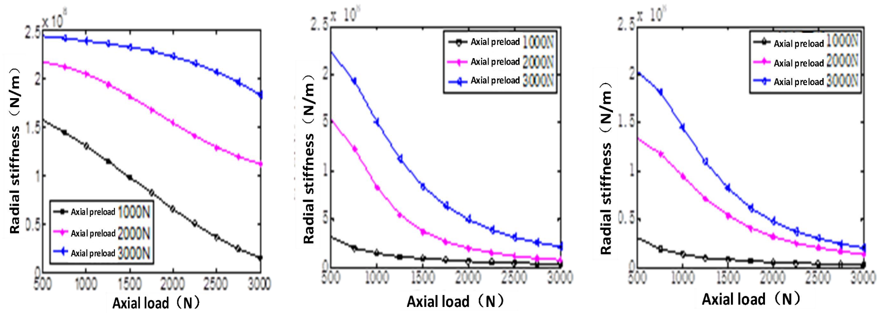
stiffness finite element results of special 7013 AC angular contact ball bearings.

4) The flange edge of the outer ring of the special-shaped bearing has little contribution to the axial displacement and axial stiffness of the bearing, and only plays the role of structural connection.

4.2 Analysis of Influence of Axial Preload on Bearing Stiffness

Bearing pre-tightening is divided into radial pre-tightening and axial pre-tightening according to the pre-tightening direction. Generally, cylindrical roller bearings are pre-tightened in radial direction, while tapered roller bearings, angular contact ball bearings, etc. are often used in axial pre-tightening. Bearing pre-tightening has a great influence on bearing stiffness and rotation accuracy, which can restrain structural vibration and prolong effective life by increasing connection reliability. The main bearing of angular contact ball for aero-engine needs to bear a large axial thrust. Proper pre-tightening can prevent the rolling element of the bearing from revolving and slipping during high-speed rotation, reduce the rolling element's spinning and sliding, reduce the friction heat generation of the bearing and prolong the service life of the bearing.

When analyzing the stiffness characteristics of the elastic support fulcrum of the rotor system, the influence of the rolling bearing on the radial stiffness is mainly considered [11], so the radial stiffness of the bearing is mainly analyzed and compared with that of the finite element method. This chapter selects 1000 N, 2000 N and 3000 N axial pre-tightening states to study the influence of axial pre-tightening on radial stiffness of angular contact ball bearings.



(a) (b) (c)

Fig.12. Radial stiffness of three kinds of bearing models under different radial loads

Where, (a) is special-shaped 7013 analytical solution,(b) is ordinary 7013 finite element solution,(c) is special-shaped 7013 finite element solution.

According to Fig.12, it can be seen that the change of radial stiffness of rolling bearings under different axial pre-tightening forces and radial loads can be seen:

1) With the increase of radial load, the radial stiffness of rolling bearing tends to decrease;

2) The greater the axial preload of the bearing, the greater the radial stiffness of the bearing under the same radial force;

3) Under the same axial pre-tightening state, the analytical calculation result of radial stiffness of ordinary angular contact ball bearings is larger than the fitting result of finite element calculation, which is in the same order of magnitude and has little difference; Under the same axial pre-tightening state, the radial stiffness of the special-shaped angular contact ball bearing is slightly smaller than that of the ordinary angular contact ball bearing, and the flange of the outer ring is deformed but relatively less than the overall deformation.

5 Conclusions

Based on the established analytical model of bearing stiffness and finite element analysis software, this paper analyzes the stiffness of common angular contact ball bearings and special angular contact ball bearings. Based on Hertz contact theory and considering the axial pre-tightening state of different bearings, the radial stiffness characteristics of common bearings and special bearings are compared and analyzed.

(1) The five-degree-of-freedom stiffness model of bearing based on Hertz contact theory and rigid ferrule hypothesis can effectively calculate the main stiffness of bearing with high accuracy, but the stiffness of bearing is smaller and stiffer due to the adoption of rigid ferrule hypothesis. The analysis model and finite element model show that the bearing stiffness changes are nonlinear.

(2) With the increase of radial load, the radial displacement of the bearing tends to increase and the radial stiffness of the bearing tends to decrease; The radial stiffness of ordinary angular contact ball bearings is smaller than the analytical results and slightly larger than the radial stiffness of special angular contact ball bearings. The flange edge of the outer ring of the special-shaped bearing does not contribute much to the radial displacement and radial stiffness of the bearing, but only plays a role of structural connection.

(3) With the increase of radial load, the radial stiffness of angular contact ball bearings under the same axial preload tends to decrease; The greater the axial preload, the greater the radial stiffness of the bearing under the same radial force. Under the same axial pre-tightening state, the analytical solution of radial stiffness of ordinary angular contact ball bearings is larger than the finite element solution, the radial stiffness of special angular contact ball bearings is slightly smaller than that of ordinary angular contact ball bearings, and the flange edge of the outer ring is deformed but relatively small in overall deformation.

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