Influence of the Change of Pre-tightening Forces State and Loosening on Modal and Vibration Response of Disc-Drum Component

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Abstract: In this paper, the finite element model and the solid model of the disc-drum simulation component are established, and the influence of the state change (loosening) of the preload on its modal and vibration response is analyzed by using the finite element method and the experimental method respectively. The simulation results show that the low-order natural frequencies and mode shapes of the disc-drum component are almost unchanged, while some high-order natural frequencies and mode shapes are sensitive to change when there is a bolt loosening at the installation side leading to the change of the pre-tightening state. In the low frequency band of 0-1000Hz, the amplitude-frequency characteristics of the axial vibration of the measuring point do not change with the variety of pre-tension state (one bolt loosening), but the phase of the measuring point has obvious change. From the hammering test results, it can be seen that in the frequency range of 0-3000Hz, the axial vibration amplitude-frequency characteristics of the measuring point are basically unchanged with the variety of the pre-tightening force state, but in the high frequency range of 3000-6400Hz, the change of the pre-tightening force state will lead to a certain offset of the amplitude-frequency curve of the measuring point at some natural frequencies. Compared with the amplitude-frequency characteristics of the vibration, the phase-frequency curve of the axial vibration of the measuring point is sensitive to the change of the pre-tightening force state. When a bolt is loosened, the phase of the measured point begins to change abruptly near the fourth-order natural frequency. With the further degradation of the pretension state, the frequency value of the change of phase begins to decrease, from near the fourth-order natural frequency to near the third-order, second-order and first-order natural frequency.

Keywords: Amplitude-frequency characteristics; Phase-frequency curve; Natural frequencies; The disc-drum component

1 Introduction

As an important component of aero-engine rotor, disc-drum component is composed of multi-stage mounting edges. Through a large number of high-performance bolts, the

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installation flanges at all levels are connected, resulting in a certain degree of discontinuity of disc-drum component, which affects the dynamic characteristics of the rotor. If the connecting bolt of the disc-drum component is in the ideal uniform pre-tightening state, the transfer function of dynamic response of the structure is basically fixed in the linear conditions. When the pre-tightening force state of the bolt changes (loosening), the stiffness of the disc-drum component will be reduced to a certain extent, which will change the transfer function of the flange surface to be connected, thus leading to the change of its modal characteristics and vibration. Usually, the main parameters for evaluating the dynamic characteristics of structures are natural frequency, mode shape, damping ratio, phase difference, stiffness and mass. Therefore, through the simulation calculation and test analysis of the dynamic characteristic parameters of the disc-drum component, the influence law of the change of the pretension state (loosening) on its modal and vibration response can be studied, and the bolt pretension state on the disc-drum component can be identified.

Scholars have carried out a series of studies on the influence of bolt pretension state change (loosening) on the dynamic characteristics of the structure. Qin Zhaoye[1] studied the influence of bolt loosening on the time-varying stiffness of the connection interface by using finite element method. Based on the dynamic theory, the local stiffness change caused by bolt loosening was deduced, and then the influence of the stiffness change caused by bolt loosening on the steady-state response of the rotor was studied. K. He [2] based on the vibration method, taking the bolt-connected pipeline structure as the object, carry out the analysis of the influence of bolt loosening on the natural frequency and vibration response of the structure. The results show that the first natural frequencies of the structure change obviously when the pre-tightening state of the bolts changes, which can be used as an index to detect the loosening of the bolt connections in the pipeline structure. In addition, for simple structures with fewer bolts, researchers have carried out a series of studies on the influence of pre-tightening state on the dynamic characteristics of structures [3-7].

Although scholars have studied the influence of pre-tightening force on the modal and vibration response of structures based on dynamics and vibration theory, most of the research objects are based on single bolt or a small number of multi-bolt plate connections or flange connections, which are usually simple in structure and only have one installation side. In this paper, the modal and vibration responses of the disc-drum component with multi-bolt connections and two mounting edges are analyzed by finite element method and experimental method. The first 60 order natural frequencies, sensitive mode shapes and the axial vibration amplitude-frequency and phase-frequency curves of the disc-drum component under different preloading conditions are obtained. The experimental and simulation results can provide a reference for identifying the pre-tightening force state of aero-engine rotor structure.

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2 Finite element model of disc-drum component

2.1 The structure of disc-drum component

According to the structural similarity and referring to the specific parameters of the test equipment, the design of the scaled disc-drum component is shown in Fig. 1. The disc-drum component consists of a short shaft disc, a drum, a long shaft disc and two brackets. There are two bolt mounting edges co-existing, and 24×M6 bolts are arranged on each side. The intermediate drum is matched with the long and short shaft disc by rabbets. 4×M10 bolts are used to connect the flange of the outer end of the long and short axle discs with the bracket, and 2×M10 bolts are used to fix the bracket on the vibration table to simulate the constrained boundary conditions.

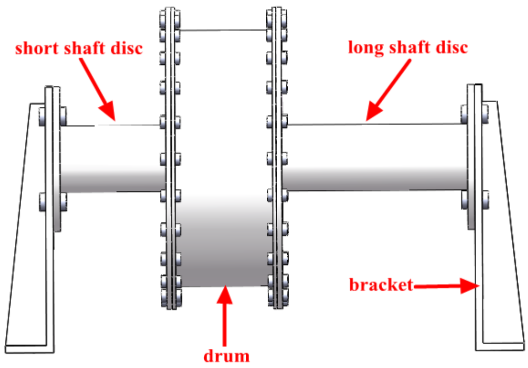


Fig. 1. Three-dimensional model of disc-drum component

2.2 Material properties

The parts of the disc-drum component are processed and manufactured with 40Cr material. The bolts are made of high strength alloy steel, and their mechanical properties are equivalent to 40Cr. The material properties of the disc-drum component are shown in Table 1, including modulus of elasticity, density and Poisson's ratio.

Table 1. Material properties of disc-drum component

Material	Modulus of elasticity/GPa	Density /Kg/m^3	Poisson's ratio
40Cr	202	7900	0.3

2.3 Contact settings

All contacts of joint surfaces and rabbets of disc-drum component are set as friction contacts, and the friction coefficient is defined as 0.15. Contact pair settings in finite element software are shown in Fig. 2.

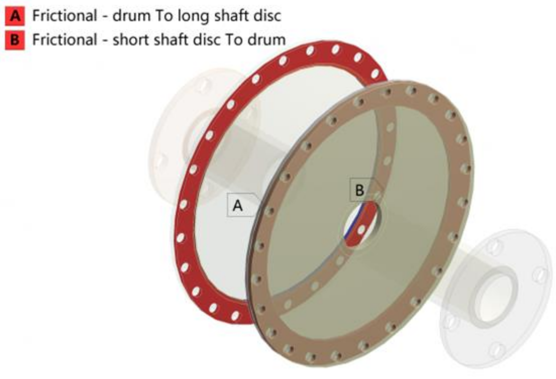
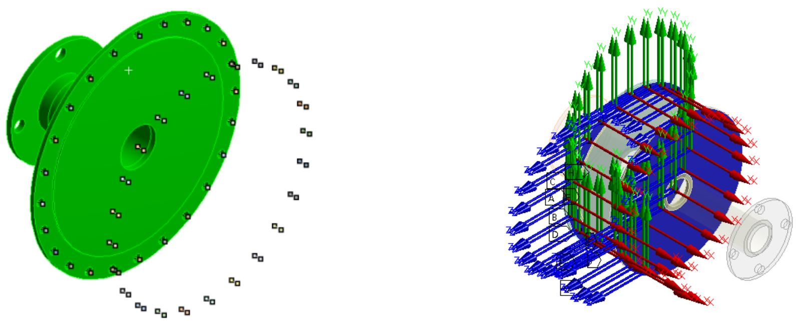


Fig. 2. Contact pair settings

2.4 Simplified model of bolt pre-tightening force

A line body is established at the center of the connecting hole between the drum and the connecting parts on both sides. It is defined as a beam element to simulate the solid bolt connection. There are 2×24 beam elements, as shown in Fig. 3 (a). The two ends of the wire bolt are coupled with the action area of the connected shaft discs by using the motion pair connection technology. Set the pinball zone control to 8mm, as shown in Fig. 3 (b). Then the preload is applied to the beam element to simulate different bolt preload states by changing the magnitude of the preload.



(a) Line bolt creation (b) Coupled settings

Fig. 3. Simplified bolt model based on beam element

The two mounting sides of the drum and the two shaft discs are connected by 24×M6 bolts. Check the table according to the bolt strength level 10.9 to determine the value of T, and according to the conversion formula T = 0.2Fd of tightening torque and pre-tightening force calculated that the pre-tightening force Fn of each bolt in the normal pre-tightening state is 8.3KN. It is set in two steps, the first step is to load the preload force value Fn = 8.3KN, and the second step is to lock it. The preload setting is shown in Fig. 4.

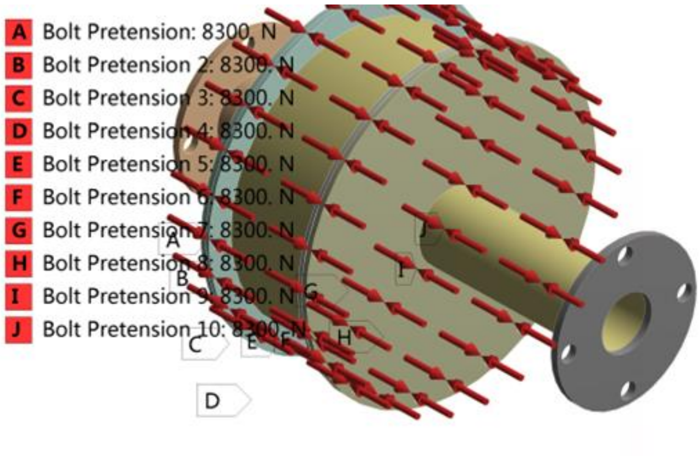
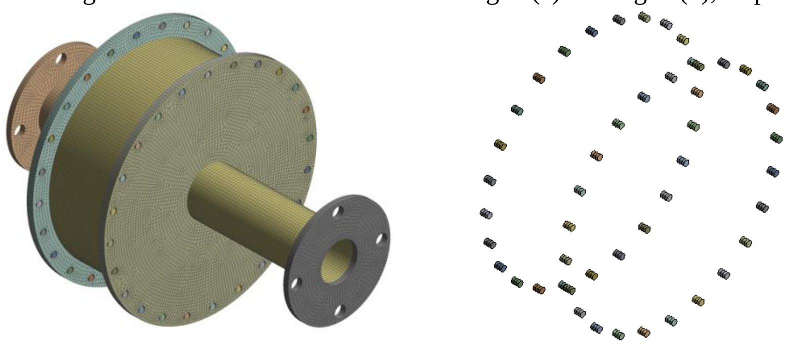


Fig. 4. Preloading settings

2.5 Element selection and mesh generation

The solid186 three-dimensional solid element in the software unit library is used in the calculation unit, which has good contact simulation ability and low calculation cost. The mesh generation method of multi-area swept hexahedron is adopted. The mesh size is set to 3 mm, the total number of mesh elements is 59102, and the number of nodes is 329 965. The average mesh quality can reach 0.91, which can meet the needs of simulation calculation. The integral mesh generation of disc-drum component model and the mesh generation of line bolt are shown in Fig. 5 (a) and Fig. 5 (b), respectively.



(a) Overall meshing (b) Meshing of line bolts

Fig. 5. Meshing

2.6 Boundary Conditions and Pre-tightening State Settings

The boundary condition of modal simulation of disc-drum component is defined as unconstrained state. The bolt number is defined as shown in Fig. 6.

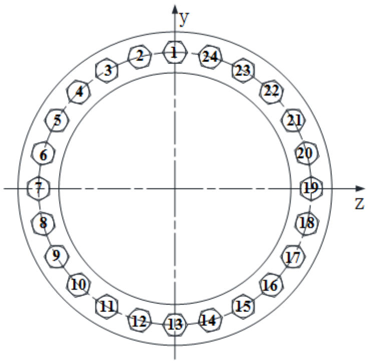


Fig. 6. The diagram of bolt number

The following two pre-tightening force states are established:

1) The bolt is in normal pre-tightening state, and the pre-tightening force Fn of each bolt is 8.3KN.

2) The 1# bolt located on the short axle disc loosened, and the pre-tightening force Fn of the other bolts is 8.3KN.

3 Analysis of calculation results

3.1 Modal analysis of disc-drum component

Firstly, the static analysis of the disc-drum component under two pre-tightening conditions is carried out by using the finite element method. Then, the modal analysis with pre-stress is started based on the static results of the analysis, and the first 60 order modes of the disc-drum component are taken for modal truncation. List the first 60 order natural frequencies as shown in Table 2. Find out the sensitive order of natural frequencies and list the corresponding mode shapes, as shown in Figure 7. From the comparison data of natural frequencies in Table 2, it can be seen that in the low frequency band of 0-3000Hz, the change of pre-tightening force state has little effect on natural frequencies. The possible reason is that when a bolt loosens, the local stiffness tends to decrease, but the influence on the overall stiffness of disc-drum component is small, while the low-order mode is mainly the overall vibration. Therefore, the low-order natural frequencies and mode shapes are insensitive to the change of the preload state. In the high frequency band above 3000 Hz, there are some orders of natural frequency changes that are sensitive. For

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example, when the preload force state changes (1 bolt is loosened), the 43th order natural frequency is reduced by 35.8 Hz, and the rate of change is 0.8%.

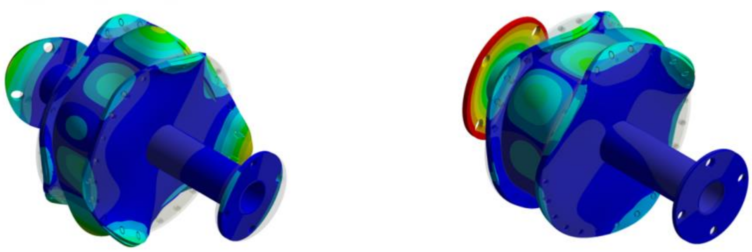
Table 2. The natural frequencies of the first 60 steps of the drum module (unit: Hz)

order	The frequencies of fully-tightened	The frequencies of 1# bolt loosening	change
1	351.17	351.05	-0.12
2	351.26	351.24	-0.02
3	465.75	465.24	-0.51
4	589.86	589.45	-0.41
5	590.18	589.79	-0.39
6	674.73	673.73	-1
7	2399.5	2399.5	0
8	2827.2	2822.3	-4.9
9	2827.3	2825.3	-2
10	2885.8	2883.9	-1.9
11	2886.3	2886.1	-0.2
12	2966.7	2964.9	-1.8
13	3027.5	3023.4	-4.1
14	3028.4	3027.5	-0.9
15	3049.9	3047.4	-2.5
16	3185.9	3185.6	-0.3
17	3197.6	3185.7	-11.9
18	3201.4	3200.5	-0.9
19	3203.8	3203.1	-0.7
20	3203.9	3203.5	-0.4
21	3213.4	3203.9	-9.5
22	3222.1	3209.6	-12.5
23	3238.2	3232.5	-5.7
24	3249.3	3242.6	-6.7
25	3412	3411.4	-0.6
26	3414.8	3414.7	-0.1
27	3447.4	3443.3	-4.1
28	3456	3451	-5
29	3497.6	3492.3	-5.3
30	3498.8	3497.1	-1.7
31	3684.9	3684.2	-0.7
32	3704.4	3689.1	-15.3
33	3730.2	3722.6	-7.6
34	3762.2	3762.1	-0.1
35	3778.7	3775.5	-3.2
36	3912	3909.9	-2.1
37	3912.9	3911.1	-1.8
38	3933.5	3931.8	-1.7
39	4014	4012	-2

40	4015.7	4014.4	-1.3
41	4146.4	4141	-5.4
42	4150.9	4149.1	-1.8

order	The frequencies of fully-tightened	The frequencies of 1# bolt loosening	change
43	4452	4416.2	-35.8
44	4454.9	4450.3	-4.6
45	4575.4	4545.1	-30.3
46	4586.1	4578.6	-7.5
47	4722.7	4721.5	-1.2
48	4722.8	4722	-0.8
49	4783.9	4777.3	-6.6
50	4784.9	4782.6	-2.3
51	4883.4	4855.8	-27.6
52	4886.4	4884.8	-1.6
53	4899.9	4891	-8.9
54	4962.9	4956.7	-6.2
55	4985.3	4966	-19.3
56	5375.6	5367.5	-8.1
57	5376.4	5375.8	-0.6
58	5484.2	5457.4	-26.8
59	5491.4	5487.4	-4
60	5527.8	5527.8	0

Further study the mode shapes corresponding to the natural frequencies of these sensitive orders, as shown in Fig. 7. It can be seen from the figure that when the pre-tension state caused by the loosening of 1 # bolt changes, the mode shapes of some sensitive orders have obvious changes. The most obvious performance is the 43th order mode. Due to the looseness of the 1# bolt, the local vibration mode is intensified at this position. Therefore, the change of pre-tightening force state of bolt can be identified by arranging measuring points and picking up vibration frequency response curve at this position.



Ideal pre-tightening state 1# bolt loosening

（a）17th order

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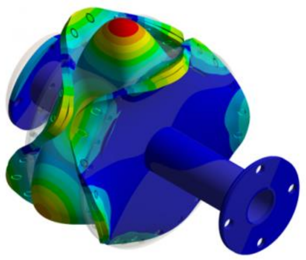
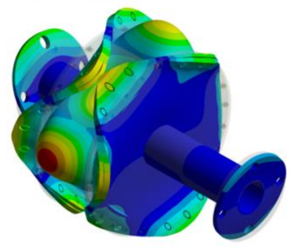
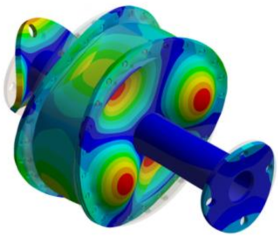
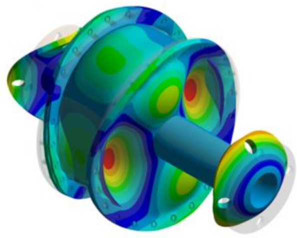



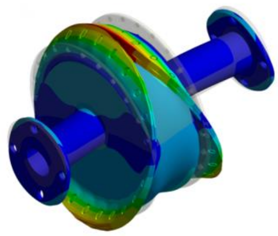
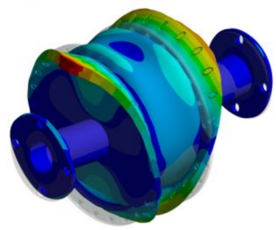
Ideal pre-tightening state　　　　1# bolt loosening

（b）22th order

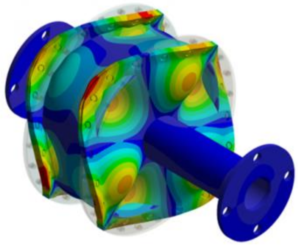
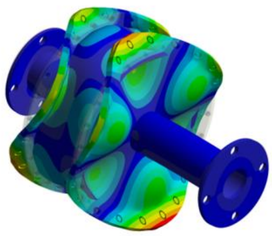
Ideal pre-tightening state　　　　1# bolt loosening

（c）32th order

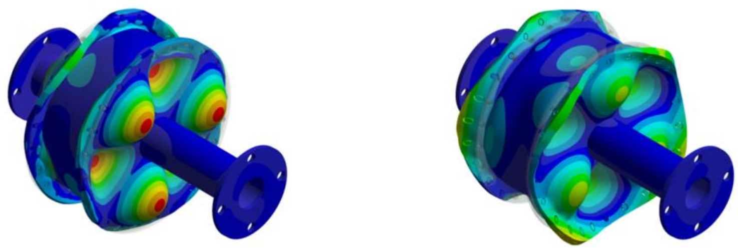
Ideal pre-tightening state　　　　1# bolt loosening

（d）43th order

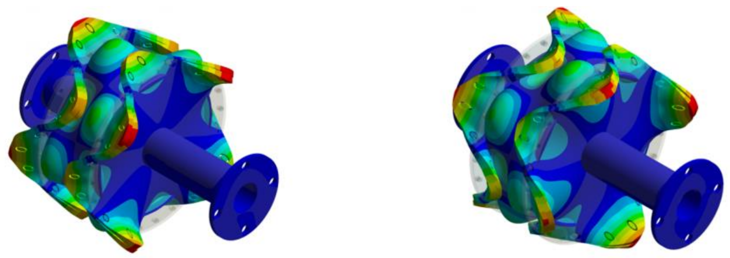
Ideal pre-tightening state　　　　1# bolt loosening

（e）45th order



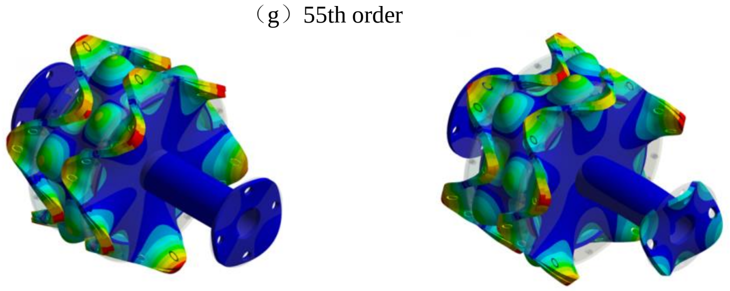
Ideal pre-tightening state 1# bolt loosening

（f）51th order



Ideal pre-tightening state 1# bolt loosening

（g）55th order



Ideal pre-tightening state 1# bolt loosening

（h）58th order

Fig. 7. Contrast nephogram of sensitive order modes of disc-drum component in two pre-tightening states

3.2 Analysis of axial vibration response of disc-drum component

Based on the modal analysis, the modal superposition method is used to analyze the harmonic response of the disc-drum component. Set the sweep frequency excitation signal to F=F0sinwt, the sweep frequency range from 0 Hz to 1000 Hz, and the interval of sweep frequency is 2 Hz. A node is defined at the position below the 1 # bolt, and the load amplitude F = 1KN is applied on the node. The loading direction is X direction. The selection of excitation point and measurement point is shown in Fig. 8.

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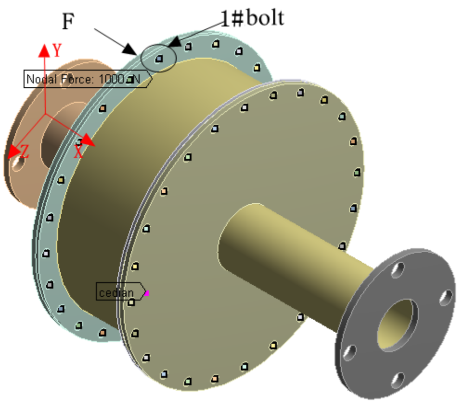
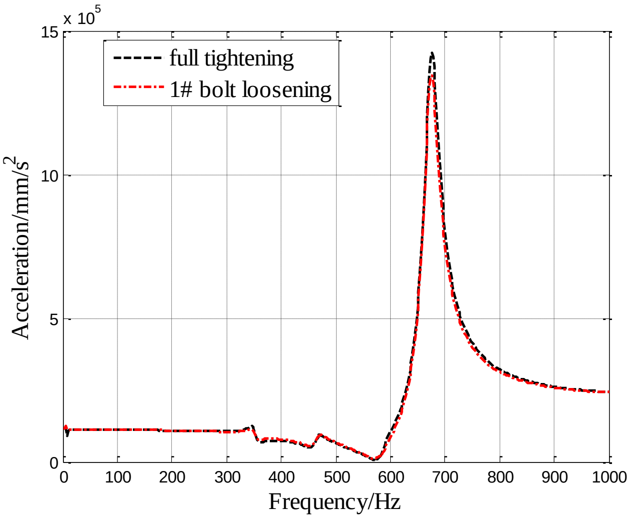
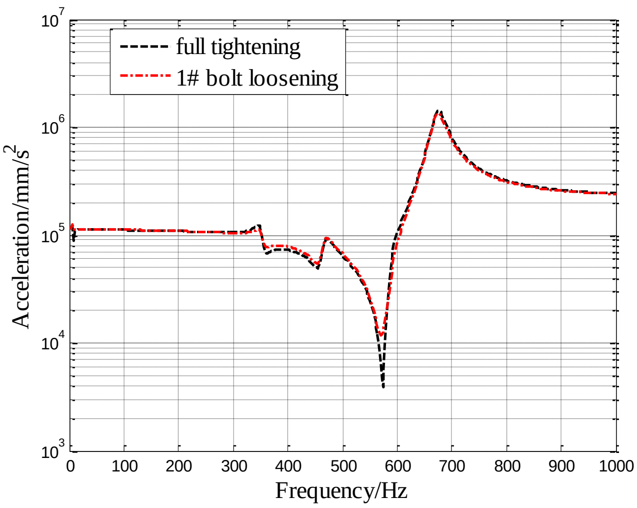


Fig. 8. Excitation point and measurement point setting

The X-direction acceleration-frequency response curves and phase-frequency curves of the measured points in the frequency band of 0Hz-1000Hz under two pre-tightening conditions are shown in Fig. 9 and Fig. 10, respectively. From Fig. 9, it can be seen that the acceleration frequency response curves of the two pre-tightening force States basically coincide in the low frequency band of 0-1000Hz, which shows that the pre-tightening force state is insensitive to the low-order acceleration amplitude-frequency response of the disc-drum component. From the phase-frequency curve shown in Fig. 10, it can be seen that when the pretension state changes (1 bolt loosening), the vibration phase of the measuring point will change significantly near the third natural frequency (590Hz).



（a）The curve of linear amplitude frequency



（b）The curve of log amplitude frequency

Fig. 9. Amplitude-frequency response curves of measuring point X-direction vibration under two pre-tightening forces in the frequency band of 0-1000Hz

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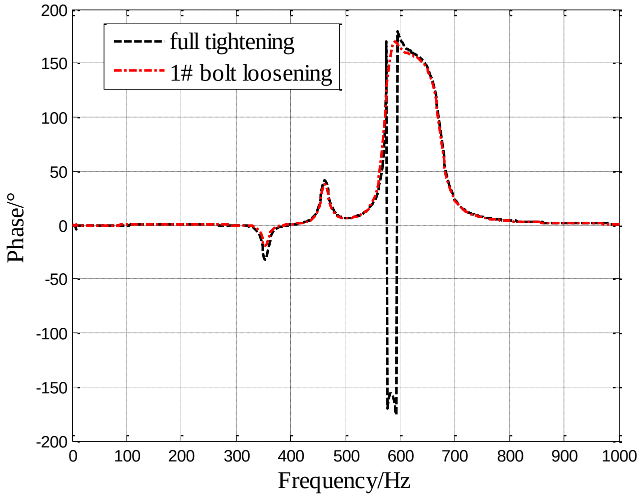

Fig. 10. Phase-frequency response curves of measuring point X-direction vibration under two pre-tightening forces in the frequency band of 0-1000Hz

4 Test analysis

4.1 Test Method

The vibration characteristics of the disc-drum component under the condition of fully fastened of bolts and 1# bolt loosening are evaluated by installing acceleration sensors. Firstly, three hammering points and two measuring points are set on the disc-drum component (two light sensors are arranged on the outer surface of the left and right flanges and distributed at 90 degrees to pick up the vibration acceleration in the X direction). The arrangement of hammer point and acceleration sensor is shown in Fig. 11. Then two preload states are defined:

State 1: Torque wrench is used to apply 10N.m torque to 48 circumferentially uniformly distributed M6 connection bolts on two mounting edges, which is called full tightening for short.

State 2: On the basis of State 1, a moment wrench is used to loosen the 1# bolt at the installation side near the end of the short shaft disc, which is abbreviated as 1# bolt loosening.

Finally, the free boundary conditions of the disc-drum component are simulated with sponge cushion support and hammering test is carried out based on LMS test software. Sampling frequency is defined as 12800 Hz, corresponding sensors and hammer point are arranged, simple signal debugging is conducted, vibration tests are carried out on the two pre-tightened states of specimen respectively, and vibration characteristic curves of the measured points are obtained. Each excitation point is hammered 3 times for averaging, and the software can directly obtain the amplitude-frequency curve and phase-frequency curve of the measuring points.

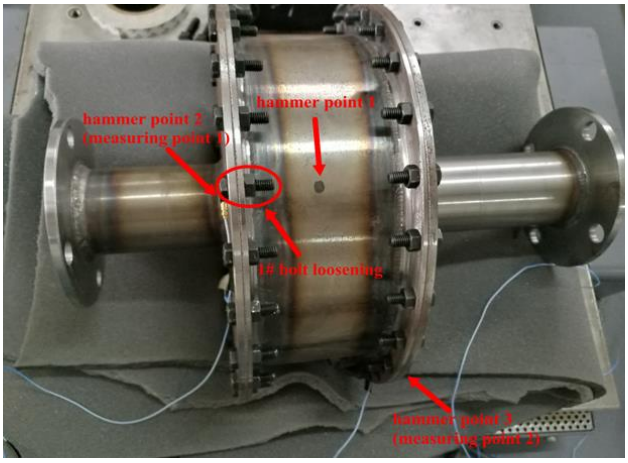

Fig. 11. The photo of hammer points and sensor arrangements

4.2 Test results

The influence of bolt pretension state change (loosening) on the axial vibration characteristics of the disc-drum component was studied by the vibration response of axial acceleration picked up by measuring point 2, excited by hammer point 2.

The amplitude-frequency and phase-frequency curves of acceleration under two pre-tightening states, which are driven by hammer point 2 and picked up by measuring point 2, are shown in Fig. 12. From the amplitude-frequency curve, it can be seen that when the pre-tightening force state changes, there is no obvious change in the 0-3000Hz frequency band, no matter linear or log chart; but in the 3000-6400Hz frequency band, the amplitude-frequency curve has a small change. From the phase-frequency curve of the measured point, it can be seen that when the pre-tightening force state changes, the

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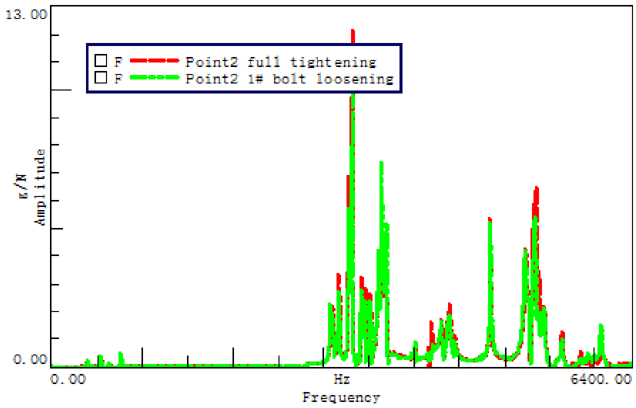
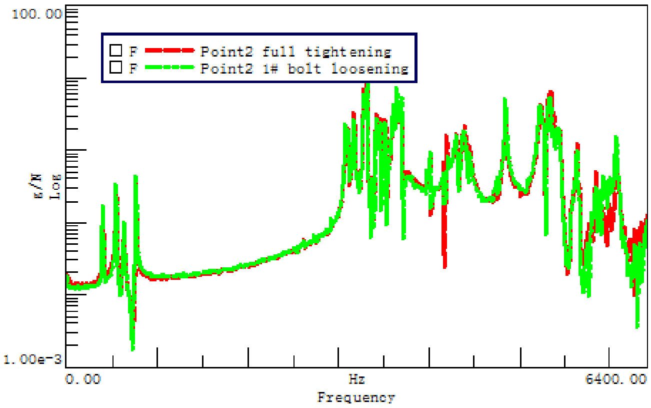
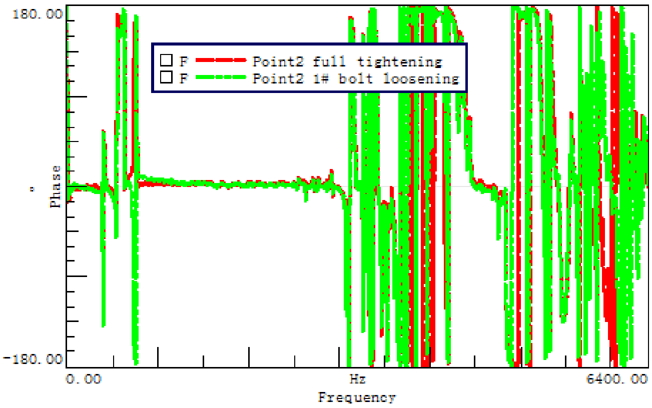
change of phase-frequency curve is more sensitive in the frequency band of 0-6400Hz. In order to further analyze the law of the amplitude and phase of the measured point's vibration in the low frequency band changing with the pre-tightening force state, the amplitude-frequency and phase-frequency curves in the 0-1000Hz band are extracted as shown in Fig. 13. It can be seen from the figure that when the pre-tightening force state changes, the amplitude-frequency response curve of the measuring point basically does not change in the low frequency band of 0-1000Hz, but the phase-frequency curve has obvious change, which is mainly manifested in the sudden change of the phase of the measuring point near the 4th order natural frequency. It shows that in the low frequency band, the amplitude-frequency response curve of the measuring point is insensitive to the change of the pre-tightening force state, while the vibration phase is sensitive to the change of the pre-tightening force state. Therefore, the vibration phase can be used as a reference index to identify the pre-tightening force state changes of the disc-drum component and even the rotor assembly.



(a) The curve of linear amplitude frequency



(b) The curve of log amplitude frequency



(c) The curve of phase frequency

Fig. 12. Acceleration amplitude-frequency phase-frequency curve of hammer point 2 excitation and measuring point 2 in two pre-tightening states

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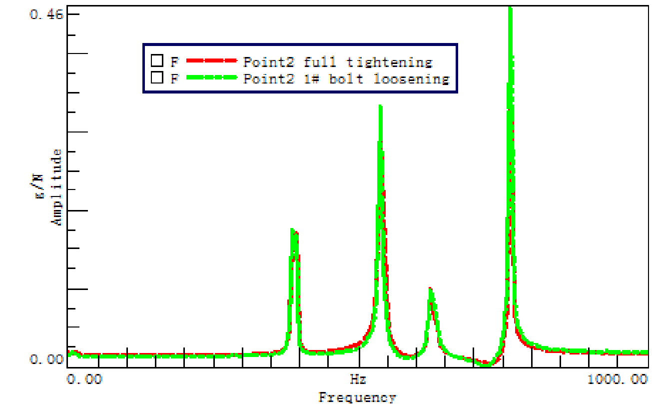
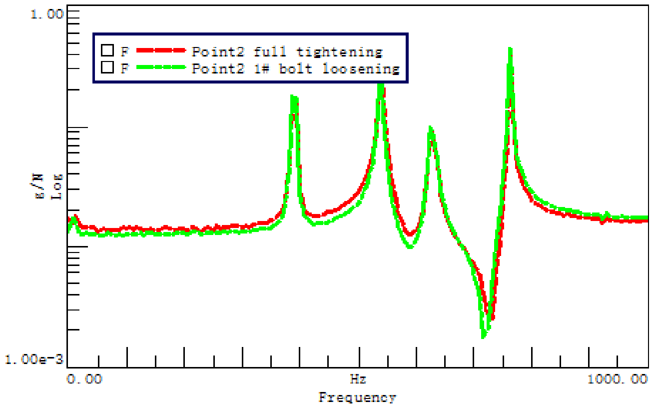
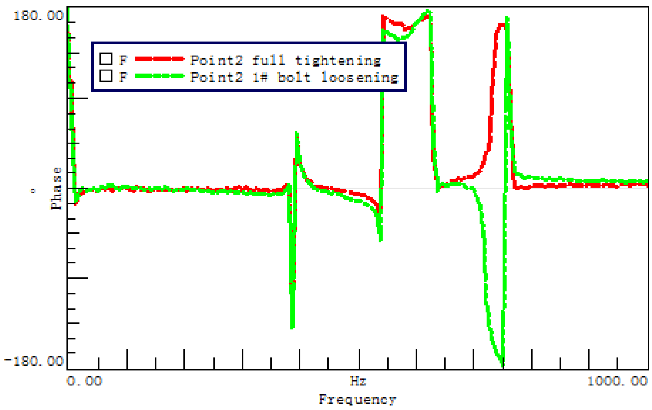


(a) The curve of linear amplitude frequency



(b) The curve of log amplitude frequency



(c) The curve of phase frequency

Fig. 13. Acceleration amplitude-frequency phase-frequency curve of the hammer point 2 excitation and the measurement point 2 in the two preloaded states in the 0-1000 Hz band

5 Comparative analysis

The first four order natural frequencies obtained by simulation and experiment are compared and analyzed, as shown in Table 3. It shows that the simulation frequency is lower than the test frequency, and the maximum error between the second-order simulation natural frequency and the test natural frequency is 15%. The reasons may be as follows: (1) the beam element is used instead of the solid bolt to simulate the pre-tightening force of the bolt, which results in the assembly stiffness of the disc-drum component being less than that of the test. (2) there are some deviations between pretension force of simulation model loading and pretension force of torque wrench loading in test.

Table 3. Comparison of the first four natural frequencies between simulation and test

Order	Simulation-frequencies（Hz）	Test-frequencies（Hz）	error（%）
1	351	393	11%
2	465	537	15%
3	589	632	7.3%
4	674	763	13%

6 Analysis of influence of pre-tightening force state on axial vibration phase of disc-drum component

From the analysis of the above chapters, it can be seen that in the low frequency band, the amplitude-frequency response curve of the measuring point is insensitive to the change of the pre-tightening force state, while the vibration phase parameter is sensitive to the change of the pre-tightening force state. In order to further study the influence of pre-tightening force state on the axial vibration phase of the disc-drum component, a variety of pre-tightening force states are set up, and the vibration test of the disc-drum component is carried out. The method of test is consistent with the test method in the previous chapter. The state of preloading force is set as follows:

State1: all bolts in the fully-tightened state

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State2: the 2# bolt (short shaft disc) is loosened

State3: the 2# bolt (short shaft disc) and 20# bolt (long shaft disc) are loosened

State4: the 1 #, 2 #, 3 # bolts (short shaft disc) and 20 # bolt (long shaft disc) are loosened

State5: the 1 #, 2 #, 3 # bolts (short shaft disc) and 19 #, 20 #, 21 # bolts (long shaft disc) are loosened

The phase-frequency curves of measuring point 2 under the excitation of hammer point 2 under various pre-tightening forces in the frequency band of 0-1000Hz are extracted, as shown in Fig. 14. From the phase-frequency curve, it can be seen that when the 2# bolt (short shaft disc) is loosened, the phase of the measuring point changed abruptly around the 4th order natural frequency. As the number of bolt loosening increases, the phase of the measuring point begins to mutate near the 3th, 2th and 1th order natural frequencies. That is to say, with the degradation of the preload state, the frequency value corresponding to the sudden change of the axial vibration phase of the disc-drum component decreases.

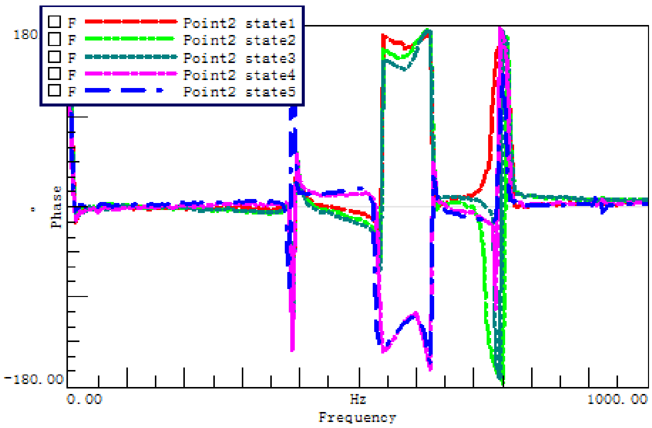

Fig. 14. Phase frequency curve of hammer point 2 at various preload conditions in the 0-1000 Hz band

7 Conclusion

Based on the results obtained from the above simulation and experimental analysis, the following conclusions can be drawn:

(1) Compared with the ideal pre-tightening state, when there is a bolt loosening at the installation side leading to the change of the pre-tightening force state, the low-order natural frequencies and mode shapes of the disc-drum component basically remain unchanged, while some higher-order natural frequencies and mode shapes have obvious changes. That is to say, the low-order modal parameters are insensitive to the slight change of the axial connection stiffness of the disc-drum component, while the high-order modal parameters are sensitive to the slight change.

(2) The simulation results of vibration response are in good agreement with the experimental results. From the amplitude-frequency response curve of the measured point obtained from the test, it can be seen that when there is a bolt loosening at the installation side which causes the change of the pre-tightening force state, the amplitude-frequency curve is basically unchanged in the frequency band of 0-3000Hz, while at the high stage (3000-6400Hz), there is a certain change, but the degree of change is low, so it is difficult to directly identify the change of axial connection stiffness of the bolt. The phase-frequency response curve of the measured point obtained from the experiment shows that when a bolt loosening causes the change of the pre-tension state, the vibration phase of the measured point begins to change abruptly near the 4th order natural frequency, which indicates that the vibration phase is sensitive to the change of the pre-tension state.

(3) The state of the preload force is changed to study the variety law of the axial vibration phase of the measuring point when the preloading state is degraded. The results show that when a bolt loosens, the phase of the measuring point begins to change abruptly near the 4th order natural frequency, and with the degradation of the preload state, the frequency value of the phase change begins to decrease, from the 4th order natural frequency to the 3th, 2th and 1th order natural frequency.

Therefore, the pre-tightening force state of the installation side can be identified by the change of high-order natural frequencies and mode shapes and the frequency value of the sudden change of the phase of the measuring point, which can provide a new idea for the dynamic assembly quality detection of the aero-engine rotor.

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